CHAPTER 3
Additional Topics in Trigonometry

CHAPTER 3
Additional Topics in Trigonometry

Section 3.1 Law of Sines

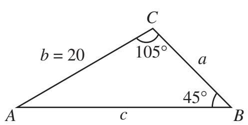
1. oblique

2. $\dfrac{b}{\sin B}$

3. angles; side

4. $\frac{1}{2}ac \sin B$

5.



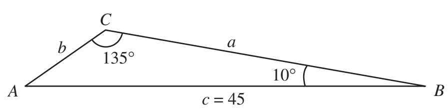
Given: $B = 45°, C = 105°, b = 20$

$A = 180° - B - C = 30°$

$a = \dfrac{b}{\sin B}(\sin A) = \dfrac{20 \sin 30°}{\sin 45°} = 10\sqrt{2} \approx 14.14$

$C = \dfrac{b}{\sin B}(\sin C) = \dfrac{20 \sin 105°}{\sin 45°} \approx 27.32$

6.



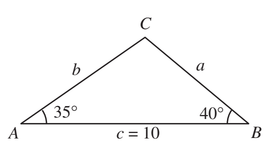
Given: $B = 10°, C = 135°, c = 45$

$A = 180° - B - C = 35°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{45 \sin 35°}{\sin 135°} \approx 36.50$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{45 \sin 10°}{\sin 135°} \approx 11.05$

7.



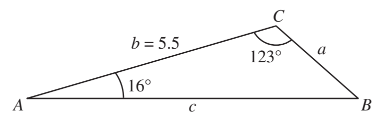
Given: $A = 35°, B = 40°, c = 10$

$C = 180° - A - B = 105°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{10 \sin 35°}{\sin 105°} \approx 5.94$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{10 \sin 40°}{\sin 105°} \approx 6.65$

8.



Given: $A = 16°, C = 123°, b = 5.5$

$B = 180° - A - C = 41°$

$a = \dfrac{b}{\sin B}(\sin A) = \dfrac{5.5}{\sin 41°}(\sin 16°) \approx 2.31$

$c = \dfrac{b}{\sin B}(\sin C) = \dfrac{5.5}{\sin 41°}(\sin 123°) \approx 7.03$

9. Given: $A = 102.4°, C = 16.7°, a = 21.6$

$B = 180° - A - C = 60.9°$

$b = \dfrac{a}{\sin A}(\sin B) = \dfrac{21.6}{\sin 102.4°}(\sin 60.9°) \approx 19.32$

$c = \dfrac{a}{\sin A}(\sin C) = \dfrac{21.6}{\sin 102.4°}(\sin 16.7°) \approx 6.36$

10. Given: $A = 24.3°, C = 54.6°, c = 2.68$

$B = 180° - A - C = 101.1°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{2.68 \sin 24.3°}{\sin 56.4°} \approx 1.35$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{2.68 \sin 101.1°}{\sin 54.6°} \approx 3.23$

11. Given: $A = 83°20', C = 54.6°, c = 18.1$

$B = 180° - A - C = 180° - 83°20' - 54°36' = 42°4'$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{18.1}{\sin 54.6°}(\sin 83°20') \approx 22.05$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{18.1}{\sin 54.6°}(\sin 42°4') \approx 14.88$

12. Given: $A = 5°40', B = 8°15', b = 4.8$

$C = 180° - A - B = 166°5'$

$a = \dfrac{b}{\sin B}(\sin A) = \dfrac{4.8 \sin 5°40'}{\sin 8°15'} \approx 3.30$

$c = \dfrac{b}{\sin B}(\sin C) = \dfrac{4.8 \sin 166°5'}{\sin 8°15'} \approx 8.05$

13. Given: $A = 35°, B = 65°, c = 10$

$$C = 180° - A - B = 80°$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{10 \sin 35°}{\sin 80°} \approx 5.82$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{10 \sin 65°}{\sin 80°} \approx 9.20$$

14. Given: $A = 120°, B = 45°, c = 16$

$$C = 180° - A - B = 15°$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{16 \sin 120°}{\sin 15°} \approx 53.54$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{16 \sin 45°}{\sin 15°} \approx 43.71$$

15. Given: $A = 55°, B = 42°, c = \frac{3}{4}$

$$C = 180° - A - B = 83°$$

$$a = \frac{c}{\sin C}(\sin A) = \frac{0.75}{\sin 83°}(\sin 55°) \approx 0.62$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{0.75}{\sin 83°}(\sin 42°) \approx 0.51$$

16. Given: $B = 28°, C = 104°, a = 3\frac{5}{8}$

$$A = 180° - B - C = 48°$$

$$b = \frac{a \sin B}{\sin A} = \frac{3\frac{5}{8} \sin 28°}{\sin 48°} \approx 2.29$$

$$c = \frac{a \sin C}{\sin A} = \frac{3\frac{5}{8} \sin 104°}{\sin 48°} \approx 4.73$$

17. Given: $A = 36°, a = 8, b = 5$

$$\sin B = \frac{b \sin A}{a} = \frac{5 \sin 36°}{8} \approx 0.36737 \Rightarrow B \approx 21.55°$$

$$C = 180° - A - B \approx 180° - 36° - 21.55 = 122.45°$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{8}{\sin 36°}(\sin 122.45°) \approx 11.49$$

18. Given: $A = 60°, a = 9, c = 7$

$$\sin C = \frac{c \sin A}{a} = \frac{7 \sin 60°}{9} \approx 0.6738 \Rightarrow C \approx 42.34°$$

$$B = 180° - A - C \approx 77.66°$$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 77.66°}{\sin 60°} \approx 10.15$$

19. Given: $A = 145°, a = 14, b = 4$

$$\sin B = \frac{b \sin A}{a} = \frac{4 \sin 145°}{14} \approx 0.1639 \Rightarrow B \approx 9.43°$$

$$C = 180° - A - B \approx 25.57°$$

$$c = \frac{a}{\sin A}(\sin C) \approx \frac{14 \sin 25.57°}{\sin 145°} \approx 10.53$$

20. Given: $A = 100°, a = 125, c = 10$

$$\sin C = \frac{c \sin A}{a} = \frac{10 \sin 100°}{125} \approx 0.07878 \Rightarrow C \approx 4.52°$$

$$B = 180° - A - C \approx 75.48°$$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{125 \sin 75.48°}{\sin 100°} \approx 122.87$$

21. Given: $B = 15°30'$, $a = 4.5$, $b = 6.8$

$$\sin A = \frac{a \sin B}{b} = \frac{4.5 \sin 15°30'}{6.8} \approx 0.17685 \Rightarrow A \approx 10°11'$$

$$C = 180° - A - B \approx 180° - 10°11' - 15°30' = 154°19'$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{6.8}{\sin 15°30'}(\sin 154°19') \approx 11.03$$

22. Given: $B = 2°45'$, $b = 6.2$, $c = 5.8$

$$\sin C = \frac{c \sin B}{b} = \frac{5.8 \sin 2°45'}{6.2} \approx 0.04488 \Rightarrow C \approx 2.57° \text{ or } 2°34'$$

$$A = 180 - B - C \approx 174.68°, \text{ or } 174°41'$$

$$a = \frac{b}{\sin B}(\sin A) \approx \frac{6.2 \sin 174.68°}{\sin 2°45'} \approx 11.99$$

23. Given: $A = 110°$, $a = 125$, $b = 100$

$$\sin B = \frac{b \sin A}{a} = \frac{100 \sin 110°}{125} \approx 0.75175 \Rightarrow B \approx 48.74°$$

$$C = 180° - A - B \approx 21.26°$$

$$c = \frac{a \sin C}{\sin A} \approx \frac{125 \sin 21.26°}{\sin 110°} \approx 48.23$$

24. Given: $a = 125$, $b = 200$, $A = 110°$

No triangle is formed because A is obtuse and $a < b$.

25. Given: $a = 18$, $b = 20$, $A = 76°$

$h = 20 \sin 76° \approx 19.41$

Because $a < h$, no triangle is formed.

26. Given: $A = 76°$, $a = 34$, $b = 21$

$$\sin B = \frac{b \sin A}{a} = \frac{21 \sin 76°}{34} \approx 0.5993 \Rightarrow B \approx 36.82°$$

$$C = 180° - A - B \approx 67.18°$$

$$c = \frac{a \sin C}{\sin A} \approx \frac{34 \sin 67.18°}{\sin 76°} \approx 32.30$$

27. Given: $A = 58°$, $a = 11.4$, $c = 12.8$

$$\sin B = \frac{b \sin A}{a} = \frac{12.8 \sin 58°}{11.4} \approx 0.9522 \Rightarrow B \approx 72.21° \text{ or } B \approx 107.79°$$

Case 1

$B \approx 72.21°$

$C = 180° - A - B \approx 49.79°$

$$c = \frac{a}{\sin A}(\sin C) \approx \frac{11.4 \sin 49.79°}{\sin 58°} \approx 10.27$$

Case 2

$B \approx 107.79°$

$C = 180° - A - B \approx 14.21°$

$$c = \frac{a}{\sin A}(\sin C) \approx \frac{11.4 \sin 14.21°}{\sin 58°} \approx 3.30$$

28. Given: $a = 4.5$, $b = 12.8$, $A = 58°$

$h = 12.8 \sin 58° \approx 10.86$

Because $a < h$, no triangle is formed.

29. Given: $A = 120°$, $a = b = 25$

No triangle is formed because A is obtuse and $a = b$.

30. Given: $A = 120°, a = 25, b = 24$

$$\sin B = \frac{b \sin A}{a} = \frac{24 \sin 120°}{25} = \frac{12\sqrt{3}}{25} \approx 0.8314 \Rightarrow B \approx 56.24°$$

$$C = 180° - A - B \approx 3.76°$$

$$c = \frac{a}{\sin A}(\sin C) \approx \frac{25 \sin 3.76°}{\sin 120°} \approx 1.89$$

31. Given: $A = 45°, a = b = 1$

Because $a = b = 1, B = 45°$.

$$C = 180° - A - B = 90°$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{1 \sin 90°}{\sin 45°} = \sqrt{2} \approx 1.41$$

32. Given: $A = 25°4', a = 9.5, b = 22$

$$\sin B = \frac{b \sin A}{a} = \frac{22 \sin 25°4'}{9.5} \approx 0.981 \Rightarrow B \approx 78.85° \text{ or } B \approx 101.15°$$

Case 1

$B \approx 78.85°$

$C = 180° - A - B \approx 76.08°$

$$c = \frac{a}{\sin A}(\sin C) \approx \frac{9.5 \sin 76.08'}{\sin 25°4'} \approx 21.76$$

Case 2

$B \approx 101.15°$

$C = 180° - A - B \approx 53.78°$

$$c = \frac{a}{\sin A}(\sin C) \approx \frac{9.5 \sin 53.78°}{\sin 25°4'} \approx 18.09$$

33. Given: $A = 36°, a = 5$

(a) One solution if $b \le 5$ or $b = \dfrac{5}{\sin 36°}$.

(b) Two solutions if $5 < b < \dfrac{5}{\sin 36°}$.

(c) No solution if $b > \dfrac{5}{\sin 36°}$.

34. Given: $A = 60°, a = 10$

(a) One solution if $b \le 10$ or $b = \dfrac{10}{\sin 60°}$.

(b) Two solutions if $10 < b < \dfrac{10}{\sin 60°}$.

(c) No solutions if $b > \dfrac{10}{\sin 60°}$.

35. Given: $A = 105°, a = 80$

(a) One solution if $b < 80$.

(b) Not possible for two solutions.

(c) No solution if $b \ge 80$.

36. Given: $A = 132°, a = 215$

(a) One solution if $b < 215$.

(b) Not possible for two solutions.

(c) No solutions if $b \ge 215$.

37. $A = 125°, b = 9, c = 6$

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}(9)(6) \sin 125° \approx 22.1$$

38. $C = 150°, a = 17, b = 10$

$$\text{Area} = \frac{1}{2}ab \sin C = \frac{1}{2}(17)(10) \sin 150° = 42.5$$

39. $B = 39°, a = 25, c = 12$

$$\text{Area} = \frac{1}{2}ac \sin B = \frac{1}{2}(25)(12) \sin 39° = 94.4$$

40. $A = 72°, b = 31, c = 44$

$$\text{Area} = \frac{1}{2}bc \sin A$$
$$= \frac{1}{2}(31)(44)\sin 72° \approx 648.6$$

41. $C = 103° 15', a = 16, b = 28$

$$\text{Area} = \frac{1}{2}ab \sin C$$
$$= \frac{1}{2}(16)(28)\sin 103° 15' \approx 218.0$$

42.
$$\text{Area} = \frac{1}{2}ac \sin B$$
$$= \frac{1}{2}(62)(35)\sin 54° 30'$$
$$= \frac{1}{2}(62)(35)\sin 54.5°$$
$$\approx 883.3$$

43. $A = 67°, B = 43°, a = 8$

$$b = \frac{a}{\sin A}(\sin B) = \frac{8 \sin 43°}{\sin 67°} \approx 5.927$$
$$C = 180° - A - B = 70°$$
$$\text{Area} = \frac{1}{2}ab \sin C$$
$$= \frac{1}{2}(8)(5.927)\sin 70° \approx 22.3$$

44. $B = 118°, C = 29°, a = 52$

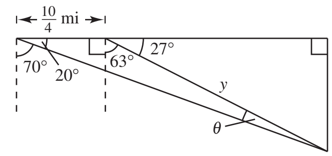
$$A = 180° - B - C = 33°$$
$$b = \frac{a}{\sin A}(\sin B) = \frac{52 \sin 118°}{\sin 33°} \approx 84.3004$$
$$\text{Area} = \frac{1}{2}ab \sin C$$
$$= \frac{1}{2}(52)(84.3004)\sin 29° \approx 1062.6$$

45. (a) $C = 180° - 94° - 30° = 56°$

$$\frac{h}{\sin 30°} = \frac{40}{\sin 56°}$$
$$h = \frac{40}{\sin 56°}(\sin 30°)$$

(b) $h = \dfrac{40}{\sin 56°}(\sin 30°) \approx 24.1$ meters

46.



In 15 minutes the boat has traveled

$$(10 \text{ mph})\left(\frac{1}{4}\text{ hr}\right) = \frac{10}{4} \text{ miles.}$$
$$\theta = 180° - 20° - (90° + 63°)$$
$$\theta = 7°$$
$$\frac{10/4}{\sin 7°} = \frac{y}{\sin 20°}$$
$$y \approx 7.0161$$
$$\sin 27° = \frac{d}{7.0161}$$
$$d \approx 3.2 \text{ miles}$$

47. Given: $A = 15°, B = 135°, c = 30$

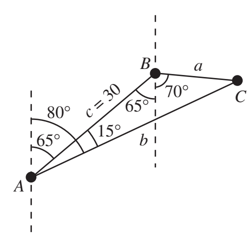
$$C = 180° - A - B = 30°$$

From Pine Knob:

$$b = \frac{c \sin B}{\sin C} = \frac{30 \sin 135°}{\sin 30°} \approx 42.4 \text{ kilometers}$$

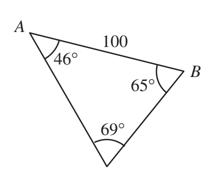
From Colt Station:

$$a = \frac{c \sin A}{\sin C} = \frac{30 \sin 15°}{\sin 30°} \approx 15.5 \text{ kilometers}$$



48. Given: $c = 100$

$$A = 74° - 28° = 46°,$$
$$B = 180° - 41° - 74° = 65°,$$
$$C = 180° - 46° - 65° = 69°$$
$$a = \frac{c}{\sin C}(\sin A) = \frac{100}{\sin 69°}(\sin 46°) \approx 77 \text{ meters}$$



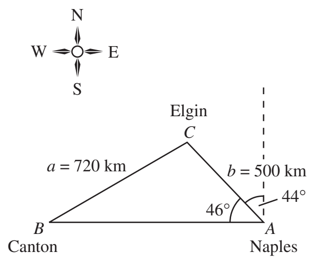
49. $\dfrac{\sin(42° - \theta)}{10} = \dfrac{\sin 48°}{17}$

$\sin(42° - \theta) \approx 0.43714$

$42° - \theta \approx 25.9°$

$\theta \approx 16.1°$

50.



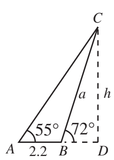
Given: $A = 46°, a = 720, b = 500$

$\sin B = \dfrac{b \sin A}{a} = \dfrac{500 \sin 46°}{720} \approx 0.50 \Rightarrow B \approx 30°$

The bearing from C to B is $240°$.

51. Given: $A = 55°, c = 2.2$

(a)



(b) $B = 180° - 72° = 108°$

$C = 180° - 55° - 108° = 17°$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin c}$

$a = \dfrac{c}{\sin c}(\sin A) = \dfrac{2.2 \sin 55°}{\sin 17°} \approx 6.16$

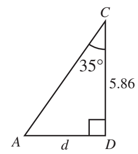
(c) $h = a \sin 72° \approx 6.16 \sin 72° \approx 5.86$ miles

(d) The plane must travel a horizontal distance d to be directly above point A.

$\angle ACD = \angle ACB + \angle BCD$

$= 17° + (180° - 72° - 90°)$

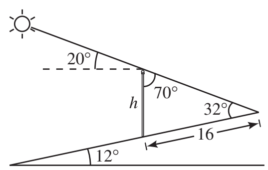
$= 17° + 18° = 35°$



$\tan 35° = \dfrac{d}{5.86}$

$d = 5.86 \tan 35° \approx 4.10$ miles

52. (a)



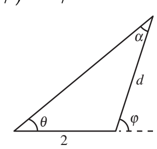
(b) $\dfrac{h}{\sin 32°} = \dfrac{16}{\sin 70°}$

(c) $h = \dfrac{16 \sin 32°}{\sin 70°} \approx 9.0$ meters

53. $\alpha = 180 - \theta - (180 - \phi) = \phi - \theta$

$\dfrac{d}{\sin \theta} = \dfrac{2}{\sin \alpha}$

$d = \dfrac{2 \sin \theta}{\sin(\phi - \theta)}$

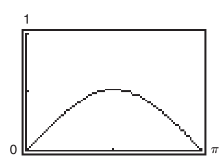


54. (a) $\dfrac{\sin \alpha}{9} = \dfrac{\sin \beta}{18}$

$\sin \alpha = 0.5 \sin \beta$

$\alpha = \arcsin(0.5 \sin \beta)$

(b)



Domain: $0 < \beta < \pi$

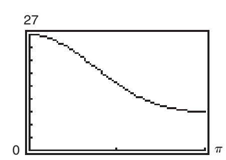
Range: $0 < \alpha \le \dfrac{\pi}{6}$

(c) $\gamma = \pi - \alpha - \beta = \pi - \beta - \arcsin(0.5 \sin \beta)$

$\dfrac{c}{\sin \gamma} = \dfrac{18}{\sin \beta}$

$c = \dfrac{18 \sin \gamma}{\sin \beta} = \dfrac{18 \sin[\pi - \beta - \arcsin(0.5 \sin \beta)]}{\sin \beta}$

(d)



Domain: $0 < \beta < \pi$

Range: $9 < c < 27$

(e)

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α	0.1960	0.3669	0.4848	0.5234	0.4720	0.3445	0.1683
c	25.95	23.07	19.19	15.33	12.29	10.31	9.27

As β increases from 0 to π, α increases and then decreases, and c decreases from 27 to 9.

55. True. If one angle of a triangle is obtuse, then there is less than $90°$ left for the other two angles, so it cannot contain a right angle. It must be oblique.

56. False. Two sides and one opposite angle do not necessarily determine a unique triangle.

57. False. To solve an oblique triangle using the Law of Sines, you need to know two angles and any side, or two sides and an angle opposite one of them.

58. True. Using the Law of Sines, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$. Dividing each side of the equation by b and multiply each side of the equation by $\sin A$, you have $\dfrac{a}{b} = \dfrac{\sin A}{\sin B}$.

59. To find the area using angle C, the formula should be $A = \dfrac{1}{2}ab \sin C$ and not $A = \dfrac{1}{2}bc \sin C$. So first find angle B, to find side a. Then the area can be calculated.

60. Distance from $(0, 0)$ to

$(4, 3)$: $\sqrt{(4 - 0)^2 + (3 - 0)^2} = 5$

A is acute.

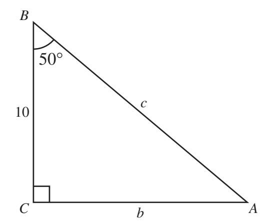
(a) $a \geq 5, a = 3$

(b) $3 < a < 5$

(c) $a < 3$

61. Yes.

$A = 180° - B - C = 40°$



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a}{\sin A} \sin C \approx 15.6$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b = \frac{c}{\sin C} \sin B \approx 11.9$$

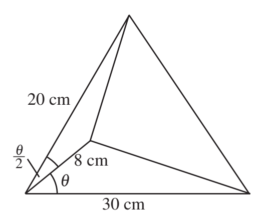
An alternative method is to use the trigonometric ratios of a right triangle.

That is $\sin A = \dfrac{opp}{hyp} = \dfrac{a}{c}$ and $\tan A = \dfrac{opp}{hyp} = \dfrac{a}{b}$.

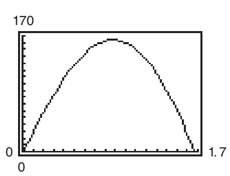
62. (a) $$A = \frac{1}{2}(30)(20)\sin\left(\theta + \frac{\theta}{2}\right) - \frac{1}{2}(8)(20)\sin\frac{\theta}{2} - \frac{1}{2}(8)(30)\sin\theta$$

$$= 300 \sin\frac{3\theta}{2} - 80 \sin\frac{\theta}{2} - 120 \sin\theta$$

$$= 20\left[15 \sin\frac{3\theta}{2} - 4 \sin\frac{\theta}{2} - 6 \sin\theta\right]$$



(b)



(c) Domain: $0 \leq \theta \leq 1.6690$

The domain would increase in length and the area would have a greater maximum value if the 8-centimeter line segment were decreased.

Section 3.2 Law of Cosines

1. $b^2 = a^2 + c^2 - 2ac \cos B$

2. alternative

3. standard

4. Heron's Area

5. Given: $a = 10, b = 12, c = 16$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{100 + 144 - 256}{2(10)(12)} = -0.05 \Rightarrow C \approx 92.87°$$

$$\sin B = \frac{b \sin C}{c} \approx \frac{12 \sin 92.87°}{16} \approx 0.749059 \Rightarrow B \approx 48.51°$$

$$A \approx 180° - 48.51° - 92.87° = 38.62°$$

6. Given: $a = 7, b = 3, c = 8$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{49 + 9 - 64}{2(7)(3)} \approx -0.142857 \Rightarrow C \approx 98.21^\circ$$

$$\sin B = \frac{b \sin C}{c} \approx \frac{3 \sin 98.21^\circ}{8} \approx 0.371157 \Rightarrow B \approx 21.79^\circ$$

$$A \approx 180^\circ - 21.79^\circ - 98.21^\circ = 60^\circ$$

7. Given: $a = 6, b = 8, c = 12$

$$\cos C = \frac{a^2 + b^2 + c^2}{2ab} = \frac{6^2 + 8^2 - 12^2}{2(6)(8)} \approx -0.458333 \Rightarrow C \approx 117.28^\circ$$

$$\sin B = b\left(\frac{\sin C}{c}\right) = 8\left(\frac{\sin 117.28^\circ}{12}\right) \approx 0.592518 \Rightarrow B \approx 36.34^\circ$$

$$A = 180^\circ - B - C \approx 180^\circ - 36.34^\circ - 117.28^\circ = 26.38^\circ$$

8. Given: $a = 9, b = 3, c = 11$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{9^2 + 3^2 - 11^2}{2(9)(3)} \approx -0.574074 \Rightarrow C \approx 125.03^\circ$$

$$\sin A = a\left(\frac{\sin C}{c}\right) \approx 9\left(\frac{\sin 125.04^\circ}{11}\right) \approx 0.669930 \Rightarrow A \approx 42.06^\circ$$

$$B = 180^\circ - A - C \approx 180^\circ - 42.06^\circ - 125.03^\circ = 12.91^\circ$$

9. Given: $A = 30^\circ, b = 15, c = 30$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$= 225 + 900 - 2(15)(30) \cos 30^\circ \approx 345.5771$$
$$a \approx 18.59$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{(345.5771)^2 + 15^2 - 30^2}{2(18.59)(15)} \approx -0.590681 \Rightarrow C \approx 126.21^\circ$$

$$B \approx 180^\circ - 30^\circ - 126.21^\circ = 23.79^\circ$$

10. Given: $C = 105^\circ, a = 9, b = 4.5$

$$c^2 = a^2 + b^2 - 2ab \cos C = 81 + 20.25 - 2(9)(4.5) \cos 105^\circ \approx 122.2143 \Rightarrow c \approx 11.06$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{81 + 122.324 - 20.25}{2(9)(11.06)} \approx 0.919598 \Rightarrow B \approx 23.13^\circ$$

$$A \approx 180^\circ - 23.13^\circ - 105^\circ = 51.87^\circ$$

11. Given: $A = 50^\circ, b = 15, c = 30$

$$a^2 = b^2 + c^2 - 2bc \cos A = 15^2 + 30^2 - 2(15)(30) \cos 50^\circ$$
$$\approx 546.4912 \Rightarrow a \approx 23.38$$

$$\sin C = c\left(\frac{\sin A}{a}\right) \approx 30\left(\frac{\sin 50^\circ}{23.3772}\right) \approx 0.983066$$

There are two angles between 0° and 180° whose sine is 0.983066, $C_1 \approx 79.4408^\circ$ and $C_2 \approx 180^\circ - 79.4408^\circ \approx 100.56^\circ$.

Because side c is the longest side of the triangle, C must be the largest angle of the triangle. So, $C \approx 100.56^\circ$ and $B = 180^\circ - A - C \approx 180^\circ - 50^\circ - 100.56^\circ = 29.44^\circ$.

12. Given: $C = 108°, a = 10, b = 7$

$$c^2 = a^2 + b^2 - 2ab\cos C = 10^2 + 7^2 - 2(10)(7)\cos 108°$$
$$\approx 192.2624 \Rightarrow c \approx 13.87$$

$$\sin A = a\left(\frac{\sin C}{c}\right) \approx 10\left(\frac{\sin 108°}{13.8659}\right) \approx 0.685896$$

There are two angles between $0°$ and $180°$ whose sine is 0.685896, $A_1 \approx 43.31°$ and $A_2 \approx 180° - 43.31° \approx 136.69°$.

Because side *c* is the longest side of the triangle, *C* is the largest angle of triangle. So, $A \approx 43.31°$ and $B = 180° - A - C \approx 180° - 43.31° - 108° \approx 28.69°$.

13. Given: $a = 11, b = 15, c = 21$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{121 + 225 - 441}{2(11)(15)} \approx -0.287879 \Rightarrow C \approx 106.73°$$

$$\sin B = \frac{b\sin C}{c} = \frac{15\sin 106.73°}{21} \approx 0.684051 \Rightarrow B \approx 43.16°$$

$$A \approx 180° - 43.16° - 106.73° = 30.11°$$

14. Given: $a = 55, b = 25, c = 72$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{55^2 + 25^2 - 72^2}{2(55)(25)} \approx -0.557818 \Rightarrow C \approx 123.90°$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{25^2 + 72^2 - 55^2}{2(25)(72)} \approx 0.773333 \Rightarrow A \approx 39.35°$$

$$B = 180° - 123.91° - 39.35° = 16.75°$$

15. Given: $a = 2.5, b = 1.8, c = 0.9$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(1.8)^2 + (0.9)^2 - (2.5)^2}{2(1.8)(0.9)} = -0.679012 \Rightarrow A \approx 132.77°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(2.5)^2 + (0.9)^2 - (1.8)^2}{2(2.5)(0.9)} \approx 0.848889 \Rightarrow B \approx 31.91°$$

$$C = 180° - 132.77° - 31.91° = 15.32°$$

16. Given: $a = 75.4, b = 52.5, c = 52.5$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{52.5^2 + 52.5^2 - 75.4^2}{2(52.5)(52.5)} = -0.031322 \Rightarrow A \approx 91.79°$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{52.5\sin(91.79°)}{75.4} \approx 0.695947 \Rightarrow B \approx 44.10°.$$

$$C = B \approx 44.10°$$

17. Given: $A = 120°, b = 6, c = 7$

$$a^2 = b^2 + c^2 - 2bc\cos A = 36 + 49 - 2(6)(7)\cos 120° = 127 \Rightarrow a \approx 11.27$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{6\sin 120°}{11.27} \approx 0.461061 \Rightarrow B \approx 27.46°$$

$$C \approx 180° - 120° - 27.46° = 32.54°$$

18. Given: $A = 48°, b = 3, c = 14$

$$a^2 = b^2 + c^2 - 2bc\cos A = 9 + 196 - 2(3)(14)\cos 48° \approx 148.7930 \Rightarrow a \approx 12.20$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{3\sin 48°}{12.20} \approx 0.182741 \Rightarrow B \approx 10.53°$$

$$C \approx 180° - 48° - 10.53° = 121.47°$$

19. Given: $B = 10°\,35', a = 40, c = 30$

$$b^2 = a^2 + c^2 - 2ac\cos B = 1600 + 900 - 2(40)(30)\cos 10°\,35' \approx 140.8268 \Rightarrow b \approx 11.87$$

$$\sin C = \frac{c\sin B}{b} = \frac{30\sin 10°\,35'}{11.87} \approx 0.464192 \Rightarrow C \approx 27.66° \approx 27°\,40'$$

$$A \approx 180° - 10°\,35' - 27°\,40' = 141°\,45'$$

20. Given: $B = 75°\,20', a = 9, c = 6$

$$b^2 = a^2 + c^2 - 2ac\cos B = (9)^2 + (6)^2 - 2(9)(6)\cos 75°\,20' \approx 89.6549 \Rightarrow b \approx 9.47$$

$$\sin A = \frac{a\sin B}{b} \approx \frac{9\sin 75°\,20'}{9.47} \approx 0.919402 \Rightarrow A \approx 66.84°, \text{ or } 66°\,51'$$

$$C \approx 180° - 75°\,20' - 66°\,51' = 37°\,49'$$

21. Given: $B = 125°\,40', a = 37, c = 37$

$$b^2 = a^2 + c^2 - 2ac\cos B = 1369 + 1369 - 2(37)(37)\cos 125°\,40' \approx 4334.4420 \Rightarrow b \approx 65.84$$

$$A = C \Rightarrow 2A = 180 - 125°\,40' = 54°\,20' \Rightarrow A = C = 27°\,10'$$

22. Given: $C = 15°\,15', a = 7.45, b = 2.15$

$$c^2 = a^2 + b^2 - 2ab\cos C = (7.45)^2 + (2.15)^2 - 2(7.45)(2.15)\cos 15°\,15' \approx 29.2180 \Rightarrow c \approx 5.41$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{(2.15)^2 + (5.41)^2 - (7.45)^2}{2(2.15)(5.41)} \approx -0.929025 \Rightarrow A \approx 158°\,17'$$

$$B \approx 180° - 158°\,17' - 15°\,15' = 6°\,28'$$

23. $C = 43°, a = \frac{4}{9}, b = \frac{7}{9}$

$$c^2 = a^2 + b^2 - 2ab\cos C = \left(\frac{4}{9}\right)^2 + \left(\frac{7}{9}\right)^2 - 2\left(\frac{4}{9}\right)\left(\frac{7}{9}\right)\cos 43° \approx 0.296842 \Rightarrow c \approx 0.54$$

$$\sin A = \frac{a\sin C}{c} = \frac{(4/9)\sin 43°}{0.544832} \approx 0.556337 \Rightarrow A \approx 33.80°$$

$$B \approx 180° - 43° - 33.8° = 103.20°$$

24. Given: $C = 101°, a = \frac{3}{8}, b = \frac{3}{4}$

$$c^2 = a^2 + b^2 - 2ab\cos C = \left(\frac{3}{8}\right)^2 + \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{8}\right)\left(\frac{3}{4}\right)\cos 101° \approx 0.8105 \Rightarrow c \approx 0.90$$

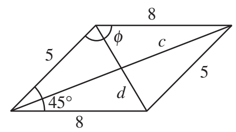
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{\left(\frac{3}{4}\right)^2 + (0.90)^2 - \left(\frac{3}{8}\right)^2}{2\left(\frac{3}{4}\right)(0.90)} \approx 0.9125 \Rightarrow A \approx 24.15°$$

$$B \approx 180° - 24.15° - 101° = 54.85°$$

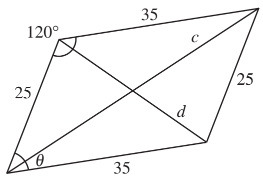
25. $d^2 = 5^2 + 8^2 - 2(5)(8)\cos 45° \approx 32.4315 \Rightarrow d \approx 5.69$

$2\phi = 360° - 2(45°) = 270° \Rightarrow \phi = 135°$

$c^2 = 5^2 + 8^2 - 2(5)(8)\cos 135° \approx 145.5685 \Rightarrow c \approx 12.07$



26.



$c^2 = 25^2 + 35^2 - 2(25)(35)\cos 120°$

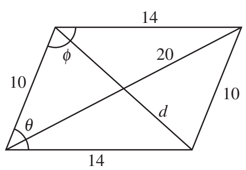
$= 2725 \Rightarrow c \approx 52.20$

$\theta = \frac{1}{2}\left[360° - 2(120°)\right] = 60°$

$d^2 = 25^2 + 35^2 - 2(25)(35)\cos 60°$

$= 975 \Rightarrow d \approx 31.22$

27.



$\cos \phi = \dfrac{10^2 + 14^2 - 20^2}{2(10)(14)}$

$\phi \approx 111.8°$

$2\theta \approx 360° - 2(111.8°)$

$\theta = 68.2°$

$d^2 = 10^2 + 14^2 - 2(10)(14)\cos 68.2°$

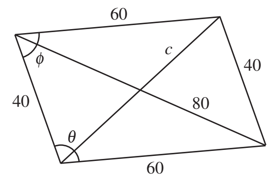
$d \approx 13.86$

28. $\cos \theta = \dfrac{40^2 + 60^2 - 80^2}{2(40)(60)} = -\dfrac{1}{4} \Rightarrow \theta \approx 104.5°$

$\phi \approx \dfrac{1}{2}\left[360° - 2(104.5°)\right] \approx 75.5°$

$c^2 \approx 40^2 + 60^2 - 2(40)(60)\cos 75.5° = 3998$

$c \approx 63.23$



29. $\cos \alpha = \dfrac{(12.5)^2 + (15)^2 - 10^2}{2(12.5)(15)} = 0.75 \Rightarrow \alpha \approx 41.41°$

$\cos \beta = \dfrac{10^2 + 15^2 - (12.5)^2}{2(10)(15)} = 0.5625 \Rightarrow \beta \approx 55.77°$

$z = 180° - \alpha - \beta = 82.82°$

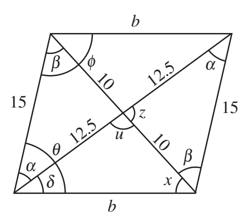
$u = 180° - z = 97.18°$

$b^2 = 12.5^2 + 10^2 - 2(12.5)(10)\cos 97.18° \approx 287.4967 \Rightarrow b \approx 16.96$

$\cos \delta = \dfrac{12.5^2 + 16.96^2 - 10^2}{2(12.5)(16.96)} \approx 0.8111 \Rightarrow \delta \approx 35.80°$

$\theta = \alpha + \delta = 41.41° + 35.80° = 77.2°$

$2\phi = 360° - 2\theta \Rightarrow \phi = \dfrac{360° - 2(77.21°)}{2} = 102.8°$



30. $\cos\alpha = \dfrac{25^2 + 17.5^2 - 25^2}{2(25)(17.5)}$

$\alpha \approx 69.512°$

$\beta \approx 180 - \alpha \approx 110.488°$

$a^2 = 17.5^2 + 25^2 - 2(17.5)(25)\cos 110.488°$

$a \approx 35.18$

$z = 180 - 2\alpha \approx 40.975°$

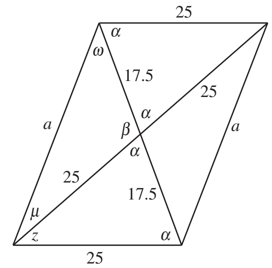
$\cos\mu = \dfrac{25^2 + 35.18^2 - 17.5^2}{2(25)(35.18)}$

$\mu \approx 27.775°$

$\theta = \mu + z \approx 68.7°$

$\omega = 180° - \mu - \beta \approx 41.738°$

$\phi = \omega + \alpha \approx 111.3°$



31. Given: $a = 8, c = 5, B = 40°$

Given two sides and included angle, use the Law of Cosines.

$b^2 = a^2 + c^2 - 2ac\cos B = 64 + 25 - 2(8)(5)\cos 40° \approx 27.7164 \Rightarrow b \approx 5.26$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \approx \dfrac{(5.26)^2 + 25 - 64}{2(5.26)(5)} \approx -0.2154 \Rightarrow A \approx 102.44°$

$C \approx 180° - 102.44° - 40° = 37.56°$

32. Given: $a = 10, b = 12, C = 70°$

Given two sides and included angle, use the Law of Cosines.

$c^2 = a^2 + b^2 - 2ab\cos C = 100 + 144 - 2(10)(12)\cos 70° \approx 161.9152 \Rightarrow c \approx 12.72$

$\sin B = \dfrac{b\sin C}{c} \approx \dfrac{12\sin 70°}{12.72} \approx 0.8865 \Rightarrow B \approx 62.44°$

$A \approx 180° - 62.44° - 70° = 47.56°$

33. Given: $A = 24°, a = 4, b = 18$

Given two sides and an angle opposite one of them, use the Law of Sines.

$h = b\sin A = 18\sin 24° \approx 7.32$

Because $a < h$, no triangle is formed.

34. Given: $a = 11, b = 13, c = 7$

Given three sides, use the Law of Cosines.

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} = \dfrac{121 + 49 - 169}{2(11)(7)} \approx 0.0065 \Rightarrow B \approx 89.63°$

$\sin A = \dfrac{a\sin B}{b} \approx \dfrac{11\sin 89.63°}{13} \approx 0.8461 \Rightarrow A \approx 57.79°$

$C \approx 180° - 57.79° - 89.63° = 32.58°$

35. Given: $A = 42°, B = 35°, c = 1.2$

Given two angles and a side, use the Law of Sines.

$$C = 180° - 42° - 35° = 103°$$

$$a = \frac{c \sin A}{\sin C} = \frac{1.2 \sin 42°}{\sin 103°} \approx 0.82$$

$$b = \frac{c \sin B}{\sin C} = \frac{1.2 \sin 35°}{\sin 103°} \approx 0.71$$

36. Given: $B = 12°, a = 160, b = 63$

Given two sides and the included angle, use the Law of Sines.

Two Solutions:

Solution #1

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin A = \frac{a \sin B}{b} = \frac{160 \sin 12°}{63} \approx 0.5280 \Rightarrow A \approx 31.87°$$

$$C = 180° - A - B$$

$$= 136.13°$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$c = \frac{b \sin c}{\sin B} = \frac{63 \sin 136.13°}{\sin 12°} \approx 210.00$$

Solution #2

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin A = \frac{a \sin B}{b} = \frac{160 \sin 12°}{63} \approx 0.5280 \Rightarrow A \approx 180° - 31.87° = 148.13°$$

$$C = 180° - A - B$$

$$= 19.87°$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$c = \frac{b \sin c}{\sin B} = \frac{63 \sin 19.8724}{\sin 12°} \approx 103.00$$

37. $a = 6, b = 12, c = 17$

$$s = \frac{a + b + c}{2} = \frac{6 + 12 + 17}{2} = 17.5$$

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{17.5(11.5)(5.5)(0.5)} \approx 23.53$$

38. $a = 33, b = 36, c = 21$

$$s = \frac{a + b + c}{2} = \frac{33 + 36 + 21}{2} = 45$$

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{45(12)(9)(24)} \approx 341.53$$

39. $a = 2.5, b = 10.2, c = 8$

$s = \frac{a + b + c}{2} = \frac{2.5 + 10.2 + 8}{2} = 10.35$

$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{10.35(7.85)(0.15)(2.35)} \approx 5.35$

40. Given: $a = 12.32, b = 8.46, c = 15.9$

$s = \frac{a + b + c}{2} = \frac{12.32 + 8.46 + 15.9}{2} = 18.34$

$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{18.34(6.02)(9.88)(2.44)} \approx 51.59$

41. Given: $a = 1, b = \frac{1}{2}, c = \frac{5}{4}$

$s = \frac{a + b + c}{2} = \frac{1 + \frac{1}{2} + \frac{5}{4}}{2} = \frac{11}{8}$

$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{\frac{11}{8}\left(\frac{3}{8}\right)\left(\frac{7}{8}\right)\left(\frac{1}{8}\right)} \approx 0.24$

42. Given: $a = \frac{3}{5}, b = \frac{4}{3}, c = \frac{7}{8}$

$s = \frac{a + b + c}{2} = \frac{\frac{3}{5} + \frac{4}{3} + \frac{7}{8}}{2} = \frac{337}{240}$

$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{\frac{337}{240}\left(\frac{193}{240}\right)\left(\frac{17}{240}\right)\left(\frac{127}{240}\right)} \approx 0.21$

43. $\text{Area} = \frac{1}{2}bc \sin A$

$= \frac{1}{2}(75)(41) \sin 80°$

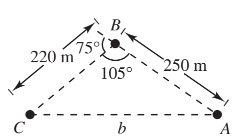
≈ 1514.14

44. $\text{Area} = \frac{1}{2}ab \sin C$

$= \frac{1}{2}(16)(3.5) \sin 109°$

≈ 26.47

45. $b^2 = 220^2 + 250^2 - 2(220)(250) \cos 105° \Rightarrow b \approx 373.3$ meters

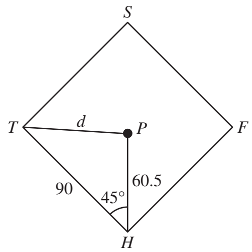


46. $\cos\theta = \frac{2^2 + 3^2 - (4.5)^2}{2(2)(3)} \approx -0.60417$

$\theta \approx 127.2°$

47. $d = \sqrt{330^2 + 420^2 - 2(330)(420) \cos 8°} \approx 103.9$ feet

48. $d^2 = 60.5^2 + 90^2 - 2(60.5)(90)\cos 45° \approx 4059.8572 \Rightarrow d \approx 63.7$ ft



49. The angles at the base of the tower are $96°$ and $84°$.

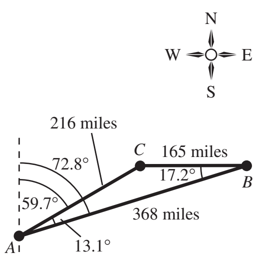
The longer guy wire g_1 is given by:

$g_1{}^2 = 75^2 + 100^2 - 2(75)(100)\cos 96° \approx 17{,}192.9 \Rightarrow g_1 \approx 131.1$ feet

The shorter guy wire g_2 is given by:

$g_2{}^2 = 75^2 + 100^2 - 2(75)(100)\cos 84 \approx 14{,}057.1 \Rightarrow g_2 \approx 118.6$ feet

50.



$a = 165, b = 216, c = 368$

$$\cos B = \frac{165^2 + 368^2 - 216^2}{2(165)(368)} \approx 0.9551$$

$B \approx 17.2°$

$$\cos A = \frac{216^2 + 368^2 - 165^2}{2(216)(368)} \approx 0.9741$$

$A \approx 13.1°$

(a) Bearing of Minneapolis (C) from Phoenix (A)

N $(90° - 17.2° - 13.1°)$ E

N 59.7° E

(b) Bearing of Albany (B) from Phoenix (A)

N $(90° - 17.2°)$ E

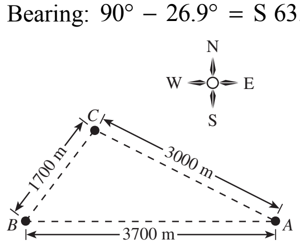
N 72.8° E

51. $\cos B = \dfrac{1700^2 + 3700^2 - 3000^2}{2(1700)(3700)} \Rightarrow B \approx 52.9°$

Bearing: $90° - 52.9° =$ N 37.1° E

$\cos C = \dfrac{1700^2 + 3000^2 - 3700^2}{2(1700)(3000)} \Rightarrow C \approx 100.2°$

Bearing: $90° - 26.9° =$ S 63.1° E



52. Distance from Franklin to Rosemount:

$d = \sqrt{810^2 + 648^2 - 2(810)(648)\cos(137°)}$

≈ 1357.8 miles

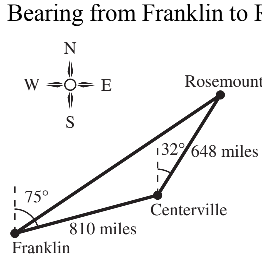
Bearing from Franklin to Rosemount:

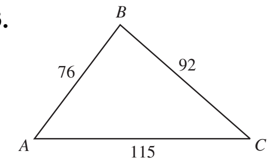
N $(75° - \theta)$ E

$$\cos\theta \approx \frac{(1357.8)^2 + 810^2 - 648^2}{2(1357.8)(810)} \approx 0.9456$$

$\theta \approx 19.0°$

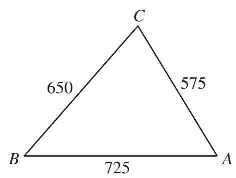
Bearing from Franklin to Rosemount: N 56.0° E



53. 

$$\cos A = \frac{115^2 + 76^2 - 92^2}{2(115)(76)} \approx 0.6028 \Rightarrow A \approx 52.9°$$

$$\cos C = \frac{115^2 + 92^2 - 76^2}{2(115)(92)} \approx 0.75203 \Rightarrow c \approx 41.2°$$

54. 

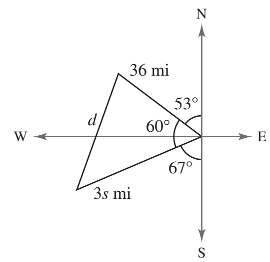
The largest angle is across from the largest side.

$$\cos C = \frac{650^2 + 575^2 - 725^2}{2(650)(575)}$$

$$C \approx 72.3°$$

55. (a) $C = 180° - 53° - 67° = 60°$

$$d^2 = a^2 + (3s)^2 - 2ab\cos C$$
$$= 36^2 + 9s^2 - 2(36)(3s)(0.5)$$
$$d = \sqrt{9s^2 - 108s + 1296}$$



(b) $43 = \sqrt{9s^2 - 108s + 1296}$

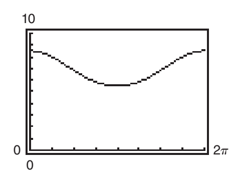
$$9s^2 - 108s - 553 = 0$$

Using the quadratic formula, $s \approx 15.87$ mph.

56. (a)
$$7^2 = 1.5^2 + x^2 - 2(1.5)x\cos\theta$$
$$49 = 2.25 + x^2 - 3x\cos\theta$$
$$x^2 - 3x\cos\theta - 46.75 = 0$$

(b) $x = \dfrac{3\cos\theta \pm \sqrt{(-3\cos\theta)^2 - 4(1)(-46.75)}}{2(1)}$

$$x = \frac{1}{2}\left(3\cos\theta + \sqrt{9\cos^2\theta + 187}\right)$$

(c) 

(d) Maximum: 8.5 inches

Minimum: 5.5 inches

In one revolution, the piston is pulled 3 inches and pushed 3 inches. The total distance the piston moves is 6 inches.

57. $a = 200$

$b = 500$

$c = 600 \Rightarrow s = \dfrac{200 + 500 + 600}{2} = 650$

Area $= \sqrt{650(450)(150)(50)} \approx 46{,}837.5$ square feet

58. Area $= 2\left[\frac{1}{2}(70)(100)\sin 70°\right]$

≈ 6577.8 square meters

(The area of the parallelogram is the sum of the areas of two triangles.)

59. $s = \dfrac{510 + 840 + 1120}{2} = 1235$

Area $= \sqrt{1235(1235 - 510)(1235 - 840)(1235 - 1120)}$

$\approx 201{,}674$ square yards

Cost $\approx \left(\dfrac{201{,}674.02}{4840}\right)(2000) \approx \$83{,}336.37$

60. Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{(a + b + c)}{2} = \frac{(2490 + 1860 + 1350)}{2} = 2850$$

Area $= \sqrt{(2850)(360)(990)(1500)} \approx 1{,}234{,}346.0 \text{ ft}^2$

$$\frac{1{,}234{,}346.0 \text{ ft}^2}{(43{,}560 \text{ ft}^2/\text{acre})} \approx 28.33669 \text{ acre}$$

$(28.33669 \text{ acre})(\$2200/\text{acre}) \approx \$62{,}340.71$

61. False. The average of the three sides of a triangle is $\dfrac{a + b + c}{3}$, not $\dfrac{a + b + c}{2} = s$.

62. False. To solve an AAS triangle, the Law of Sines is needed.

63. $c^2 = a^2 + b^2 - 2ab\cos C$

$= a^2 + b^2 - 2ab\cos 90°$

$= a^2 + b^2 - 2ab(0)$

$= a^2 + b^2$

When $C = 90°$, you obtain the Pythagorean Theorem. The Pythagorean Theorem is a special case of the Law of Cosines.

64. To solve the triangle using the Law of Cosines, substitute values into $a^2 = b^2 + c^2 - 2bc\cos A$. Simplify the equation so that you have a quadratic equation in terms of *c*. Then, find the two values of *c*, and find the two triangles that model the given information.

Using the Law of Sines will give the same result as using the Law of Cosines.

Sample answer: An advantage for using the Law of Cosines is that it is easier to choose the correct value to avoid the ambiguous case, but its disadvantage is that there are more computations. The opposite is true for the Law of Sines.

65. There is no method that can be used to solve the no-solution case of SSA.

The Law of Cosines can be used to solve the single-solution case of SSA. You can substitute values into $a^2 = b^2 + c^2 - 2bc\cos A$. The simplified quadratic equation in terms of *c* can be solved, with one positive solution and one negative solution. The negative solution can be discarded because length is positive. You can use the positive solution to solve the triangle.

66. (a) Because SSS is given, use the Law of Cosines.

(b) Because AAS is given, use the Law of Sines.

67. (a) $\frac{1}{2}bc(1 + \cos A) = \frac{1}{2}bc\left[1 + \frac{b^2 + c^2 - a^2}{2bc}\right]$

$= \frac{1}{2}bc\left[\frac{2bc + b^2 + c^2 - a^2}{2bc}\right]$

$= \frac{1}{4}\left[(b + c)^2 - a^2\right]$

$= \frac{1}{4}\left[(b + c) + a\right]\left[(b + c) - a\right]$

$= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2}$

$= \frac{a + b + c}{2} \cdot \frac{-a + b + c}{2}$

(b) $\frac{1}{2}bc(1 - \cos A) = \frac{1}{2}bc\left[1 + \frac{a^2 - (b^2 + c^2)}{2bc}\right]$

$= \frac{1}{2}bc\left[\frac{2bc + a^2 - b^2 - c^2}{2bc}\right]$

$= \frac{a^2 - (b^2 - 2bc + c^2)}{4}$

$= \frac{a^2 - (b - c)^2}{4}$

$= \left(\frac{a - (b - c)}{2}\right)\left(\frac{a + (b - c)}{2}\right)$

$= \frac{a - b + c}{2} \cdot \frac{a + b - c}{2}$

Section 3.3 Vectors in the Plane

1. directed line segment

2. initial; terminal

3. vector

4. magnitude; direction

5. standard position

6. unit vector

7. multiplication; addition

8. linear combination; horizontal; vertical

9. $\|\mathbf{u}\| = \sqrt{(6 - 2)^2 + (5 - 4)^2} = \sqrt{17}$

$\|\mathbf{v}\| = \sqrt{(4 - 0)^2 + (1 - 0)^2} = \sqrt{17}$

$\text{slope}_{\mathbf{u}} = \frac{5 - 4}{6 - 2} = \frac{1}{4}$

$\text{slope}_{\mathbf{v}} = \frac{1 - 0}{4 - 0} = \frac{1}{4}$

u and **v** have the same magnitude and direction so they are equivalent.

10. $\|\mathbf{u}\| = \sqrt{(-3-0)^2 + (-4-4)^2} = \sqrt{73}$

$\|\mathbf{v}\| = \sqrt{(0-3)^2 + (-4-3)^2} = \sqrt{58}$

$\text{slope}_{\mathbf{u}} = \dfrac{-4-4}{-3-0} = \dfrac{8}{3}$

$\text{slope}_{\mathbf{v}} = \dfrac{-4-3}{0-3} = \dfrac{7}{3}$

u and **v** do not have the same magnitude and direction, so they are not equivalent.

11. $\|\mathbf{u}\| = \sqrt{(-1-2)^2 + (4-2)^2} = \sqrt{13}$

$\|\mathbf{v}\| = \sqrt{(-5-(-3))^2 + (2-(-1))^2} = \sqrt{13}$

$\text{slope}_{\mathbf{u}} = \dfrac{4-2}{-1-2} = -\dfrac{2}{3}$

$\text{slope}_{\mathbf{v}} = \dfrac{2-(-1)}{-5-(-3)} = -\dfrac{3}{2}$

u and **v** have the same magnitude but not the same direction so they are not equivalent.

12. $\|\mathbf{u}\| = \sqrt{(7-2)^2 + (4-0)^2} = \sqrt{41}$

$\|\mathbf{v}\| = \sqrt{(2-(-8))^2 + (9-1)^2} = \sqrt{164} = 2\sqrt{41}$

$\text{slope}_{\mathbf{u}} = \dfrac{4-0}{7-2} = \dfrac{4}{5}$

$\text{slope}_{\mathbf{v}} = \dfrac{9-1}{2-(-8)} = \dfrac{4}{5}$

u and **v** have the same direction but not the same magnitude, so they are not equivalent.

13. $\|\mathbf{u}\| = \sqrt{(5-2)^2 + (-10-(-1))^2} = \sqrt{90} = 3\sqrt{10}$

$\|\mathbf{v}\| = \sqrt{(9-6)^2 + (-8-1)^2} = \sqrt{90} = 3\sqrt{10}$

$\text{slope}_{\mathbf{u}} = \dfrac{-10-(-1)}{5-2} = -3$

$\text{slope}_{\mathbf{v}} = \dfrac{-8-1}{9-6} = -3$

u and **v** have the same magnitude and direction so they are equivalent.

14. $\|\mathbf{u}\| = \sqrt{(13-8)^2 + (-1-1)^2} = \sqrt{29}$

$\|\mathbf{v}\| = \sqrt{(-7-(-2))^2 + (6-4)^2} = \sqrt{29}$

$\text{slope}_{\mathbf{u}} = \dfrac{-1-1}{13-8} = -\dfrac{2}{5}$

$\text{slope}_{\mathbf{v}} = \dfrac{6-4}{-7-(-2)} = -\dfrac{2}{5}$

u and **v** have the same magnitude and direction so they are equivalent.

15. Initial point: $(0, 0)$

Terminal point: $(1, 3)$

$\mathbf{v} = \langle 1-0, 3-0\rangle = \langle 1, 3\rangle$

$\|\mathbf{v}\| = \sqrt{1^2 + 3^2} = \sqrt{10}$

16. Initial point: $(0, 0)$

Terminal point: $(-4, -2)$

$\mathbf{v} = \langle -4-0, -2-0\rangle = \langle -4, -2\rangle$

$\|\mathbf{v}\| = \sqrt{(-4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$

17. Initial point: $(3, -2)$

Terminal point: $(3, 3)$

$\mathbf{v} = \langle 3-3, 3-(-2)\rangle = \langle 0, 5\rangle$

$\|\mathbf{v}\| = \sqrt{0^2 + 5^2} = \sqrt{25} = 5$

18. Initial point: $(-4, -1)$

Terminal point: $(3, -1)$

$\mathbf{v} = \langle 3-(-4), -1-(-1)\rangle = \langle 7, 0\rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + 0^2} = 7$

19. Initial point: $(-3, -5)$

Terminal point: $(-11, 1)$

$\mathbf{v} = \langle -11-(-3), 1-(-5)\rangle = \langle -8, 6\rangle$

$\|\mathbf{v}\| = \sqrt{(-8)^2 + 6^2} = \sqrt{100} = 10$

20. Initial point: $(-2, 7)$

Terminal point: $(5, -17)$

$\mathbf{v} = \langle 5-(-2), -17-7\rangle = \langle 7, -24\rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + (-24)^2} = 25$

21. Initial point: $(1, 3)$

Terminal point: $(-8, -9)$

$\mathbf{v} = \langle -8-1, -9-3\rangle = \langle -9, -12\rangle$

$\|\mathbf{v}\| = \sqrt{(-9)^2 + (-12)^2} = \sqrt{225} = 15$

22. Initial point: $(17, -5)$

Terminal point: $(9, 3)$

$\mathbf{v} = \langle 9-17, 3-(-5)\rangle = \langle -8, 8\rangle$

$\|\mathbf{v}\| = \sqrt{(-8)^2 + 8^2} = 8\sqrt{2}$

23. Initial point: $(-1, 5)$

Terminal point: $(15, -21)$

$\mathbf{v} = \langle 15 - (-1), -21 - 5\rangle = \langle 16, -26\rangle$

$\|\mathbf{v}\| = \sqrt{(16)^2 + (-26)^2} = \sqrt{932} = 2\sqrt{233}$

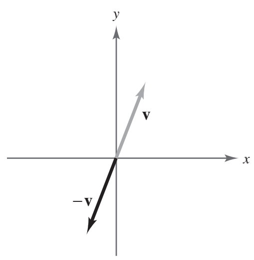
24. Initial point: $(-3, 11)$

Terminal point: $(9, 40)$

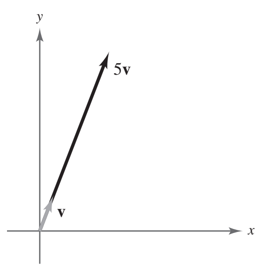
$\mathbf{v} = \langle 9 - (-3), 40 - 11\rangle = \langle 12, 29\rangle$

$\|\mathbf{v}\| = \sqrt{12^2 + 29^2} = \sqrt{985}$

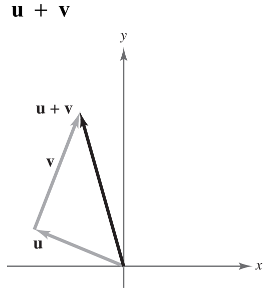
25. $-\mathbf{v}$



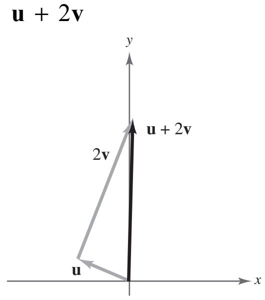
26. $5\mathbf{v}$



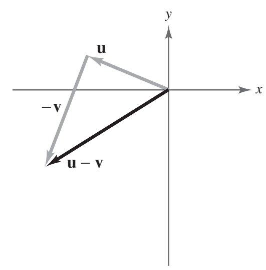
27. $\mathbf{u} + \mathbf{v}$



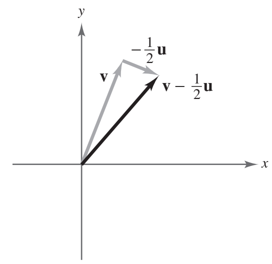
28. $\mathbf{u} + 2\mathbf{v}$



29. $\mathbf{u} - \mathbf{v}$

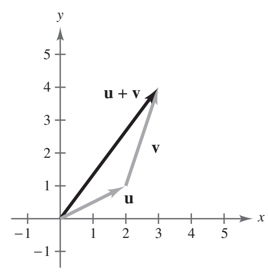


30. $\mathbf{v} - \frac{1}{2}\mathbf{u}$

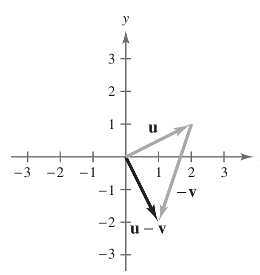


31. $\mathbf{u} = \langle 2, 1\rangle, \mathbf{v} = \langle 1, 3\rangle$

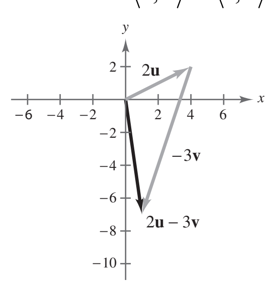
(a) $\mathbf{u} + \mathbf{v} = \langle 3, 4\rangle$



(b) $\mathbf{u} - \mathbf{v} = \langle 1, -2\rangle$

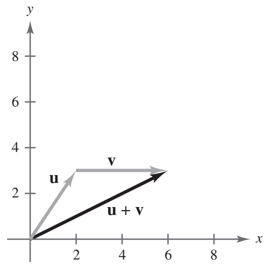


(c) $2\mathbf{u} - 3\mathbf{v} = \langle 4, 2\rangle - \langle 3, 9\rangle = \langle 1, -7\rangle$

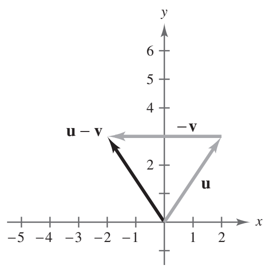


32. $\mathbf{u} = \langle 2, 3\rangle, \mathbf{v} = \langle 4, 0\rangle$

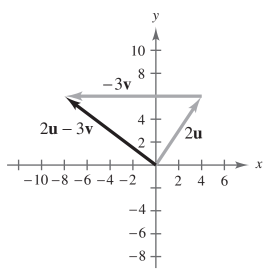
(a) $\mathbf{u} + \mathbf{v} = \langle 6, 3\rangle$



(b) $\mathbf{u} - \mathbf{v} = \langle -2, 3\rangle$

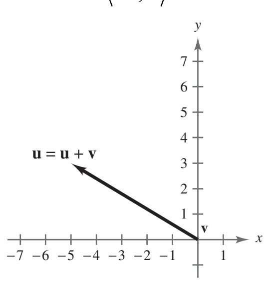


(c) $2\mathbf{u} - 3\mathbf{v} = \langle 4, 6\rangle - \langle 12, 0\rangle$
$= \langle -8, 6\rangle$

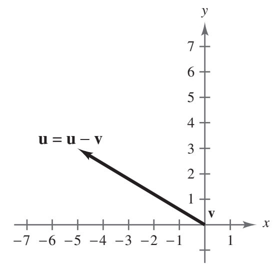


33. $\mathbf{u} = \langle -5, 3\rangle, \mathbf{v} = \langle 0, 0\rangle$

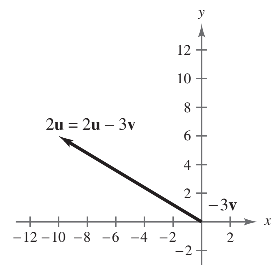
(a) $\mathbf{u} + \mathbf{v} = \langle -5, 3\rangle = \mathbf{u}$



(b) $\mathbf{u} - \mathbf{v} = \langle -5, 3\rangle = \mathbf{u}$

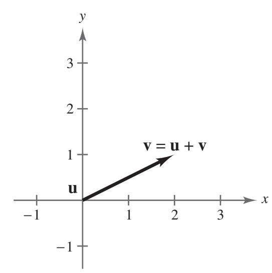


(c) $2\mathbf{u} - 3\mathbf{v} = \langle -10, 6\rangle = 2\mathbf{u}$

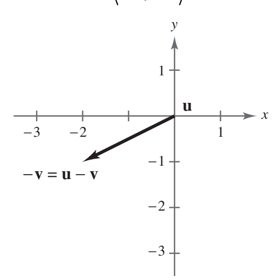


34. $\mathbf{u} = \langle 0, 0\rangle, \mathbf{v} = \langle 2, 1\rangle$

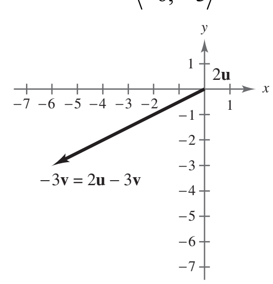
(a) $\mathbf{u} + \mathbf{v} = \langle 2, 1\rangle$



(b) $\mathbf{u} - \mathbf{v} = \langle -2, -1\rangle$



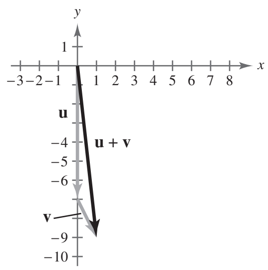
(c) $2\mathbf{u} - 3\mathbf{v} = \langle 0, 0\rangle - \langle 6, 3\rangle$
$= \langle -6, -3\rangle$



35. $\mathbf{u} = -7\mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

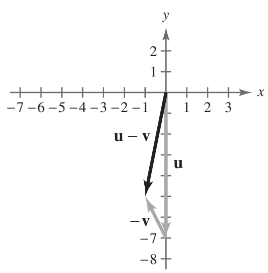
(a) $\mathbf{u} + \mathbf{v} = \mathbf{i} - 9\mathbf{j}$

$\langle 1, -9 \rangle$



(b) $\mathbf{u} - \mathbf{v} = -\mathbf{i} - 5\mathbf{j}$

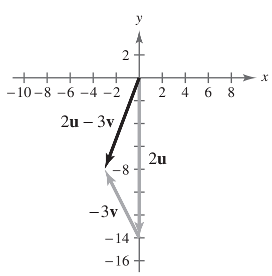
$\langle -1, -5 \rangle$



(c) $2\mathbf{u} - 3\mathbf{v} = (-14\mathbf{j}) - (3\mathbf{i} - 6\mathbf{j})$

$= -3\mathbf{i} - 8\mathbf{j}$

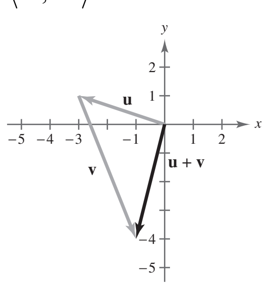
$\langle -3, -8 \rangle$



36. $\mathbf{u} = -3\mathbf{i} + \mathbf{j}, \mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$

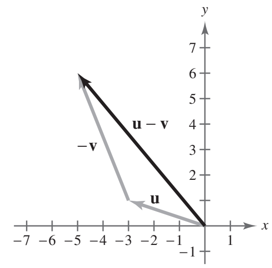
(a) $\mathbf{u} + \mathbf{v} = -\mathbf{i} - 4\mathbf{j}$

$\langle -1, -4 \rangle$



(b) $\mathbf{u} - \mathbf{v} = -5\mathbf{i} + 6\mathbf{j}$

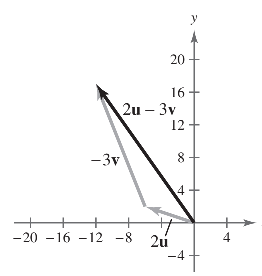
$\langle -5, 6 \rangle$



(c) $2\mathbf{u} - 3\mathbf{v} = (-6\mathbf{i} + 2\mathbf{j}) - (6\mathbf{i} - 15\mathbf{j})$

$= -12\mathbf{i} + 17\mathbf{j}$

$\langle -12, 17 \rangle$



37. $\mathbf{u} = \langle 2, 0 \rangle$

$5\mathbf{u} = \langle 10, 0 \rangle$

$\|5\mathbf{u}\| = \sqrt{(10)^2 + 0^2} = 10$

38. $\mathbf{v} = \langle -3, 6 \rangle$

$4\mathbf{v} = \langle -12, 24 \rangle$

$\|4\mathbf{v}\| = \sqrt{(-12)^4 + 24^2} = \sqrt{720} = 12\sqrt{5}$

39. $\mathbf{v} = \langle -3, 6 \rangle$

$-3\mathbf{v} = \langle 9, -18 \rangle$

$\|4\mathbf{v}\| = \sqrt{9^2 + (-18)^2} = \sqrt{405} = 9\sqrt{5}$

40. $\mathbf{u} = \langle 2, 0 \rangle$

$-\frac{3}{4}\mathbf{u} = \left\langle -\frac{3}{2}, 0 \right\rangle$

$\left\| -\frac{3}{4}\mathbf{u} \right\| = \sqrt{\left(-\frac{3}{2}\right)^2 + 0^2} = \frac{3}{2}$

41. $\mathbf{v} = \langle 3, 0 \rangle$

$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{3^2 + 0^2}}\langle 3, 0 \rangle = \frac{1}{3}\langle 3, 0 \rangle = \langle 1, 0 \rangle$

$\|\mathbf{u}\| = \sqrt{1^2 + 0^2} = 1$

42. $\mathbf{v} = \langle 0, -2\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{0^2 + (-2)^2}}\langle 0, -2\rangle = \frac{1}{2}\langle 0, -2\rangle = \langle 0, -1\rangle$$

$$\|\mathbf{u}\| = \sqrt{0^2 + (-1)^2} = 1$$

43. $\mathbf{v} = \langle -2, 2\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{(-2)^2 + 2^2}}\langle -2, 2\rangle = \frac{1}{2\sqrt{2}}\langle -2, 2\rangle$$

$$= \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$$

$$= \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$$

$$\|\mathbf{u}\| = \sqrt{\left(\frac{-\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$$

44. $\mathbf{v} = \langle -5, 12\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{(-5)^2 + 12^2}}\langle -5, 12\rangle$$

$$= \frac{1}{13}\langle -5, 12\rangle$$

$$= \left\langle -\frac{5}{13}, \frac{12}{13}\right\rangle$$

$$\|\mathbf{u}\| = \sqrt{\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = 1$$

45. $\mathbf{v} = \langle 1, -6\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{1^2 + (-6)^2}}\langle 1, -6\rangle = \frac{1}{\sqrt{37}}\langle 1, -6\rangle$$

$$= \frac{1}{\sqrt{37}}\langle 1, -6\rangle = \left\langle \frac{\sqrt{37}}{37}, -\frac{6\sqrt{37}}{37}\right\rangle$$

$$\|\mathbf{u}\| = \sqrt{\left(\frac{\sqrt{37}}{37}\right)^2 + \left(\frac{-6\sqrt{37}}{37}\right)^2} = 1$$

46. $\mathbf{v} = \langle -8, -4\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{(-8)^2 + (-4)^2}}\langle -8, -4\rangle = \frac{1}{\sqrt{80}}\langle -8, -4\rangle = \frac{1}{4\sqrt{5}}\langle -8, -4\rangle$$

$$= \frac{\sqrt{5}}{20}\langle -8, -4\rangle = \left\langle -\frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}\right\rangle$$

$$\|\mathbf{u}\| = \sqrt{\left(-\frac{2\sqrt{5}}{4}\right)^2 + \left(-\frac{\sqrt{5}}{5}\right)^2} = 1$$

47. $\mathbf{v} = 10\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 10\left(\frac{1}{\sqrt{(-3)^2 + 4^2}}\langle -3, 4\rangle\right)$

$$= 2\langle -3, 4\rangle$$

$$= \langle -6, 8\rangle$$

48. $\mathbf{v} = 3\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 3\left(\frac{1}{\sqrt{(-12)^2 + (-5)^2}}\langle -12, -5\rangle\right)$

$$= \frac{3}{13}\langle -12, -5\rangle$$

$$= \left\langle -\frac{36}{13}, -\frac{15}{13}\right\rangle$$

49. $9\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 9\left(\frac{1}{\sqrt{2^2 + 5^2}}\langle 2, 5\rangle\right) = \frac{9}{\sqrt{29}}\langle 2, 5\rangle$

$= \left\langle \frac{18}{\sqrt{29}}, \frac{45}{\sqrt{29}} \right\rangle = \left\langle \frac{18\sqrt{29}}{29}, \frac{45\sqrt{29}}{29} \right\rangle$

50. $\mathbf{v} = 8\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 8\left(\frac{1}{\sqrt{3^2 + 3^2}}\langle 3, 3\rangle\right)$

$= \frac{8}{3\sqrt{2}}\langle 3, 3\rangle$

$= \left\langle \frac{8}{\sqrt{2}}, \frac{8}{\sqrt{2}} \right\rangle$

$= \left\langle 4\sqrt{2}, 4\sqrt{2} \right\rangle$

51. $\mathbf{u} = \langle 3 - (-2), -2 - 1\rangle$

$= \langle 5, -3\rangle$

$= 5\mathbf{i} - 3\mathbf{j}$

52. $\mathbf{u} = \langle 3 - 0, 6 - (-2)\rangle$

$\mathbf{u} = \langle 3, 8\rangle$

$\mathbf{u} = 3\mathbf{i} + 8\mathbf{j}$

53. $\mathbf{u} = \langle -6 - 0, 4 - 1\rangle$

$\mathbf{u} = \langle -6, 3\rangle$

$\mathbf{u} = -6\mathbf{i} + 3\mathbf{j}$

54. $\mathbf{u} = \langle -1 - 2, -5 - 3\rangle$

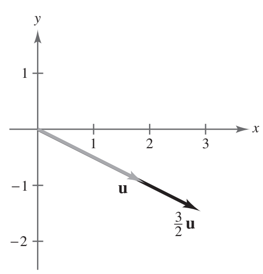
$= \langle -3, -8\rangle$

$= -3\mathbf{i} - 8\mathbf{j}$

55. $\mathbf{v} = \frac{3}{2}\mathbf{u}$

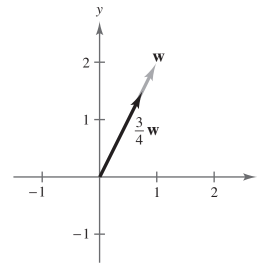
$= \frac{3}{2}(2\mathbf{i} - \mathbf{j})$

$= 3\mathbf{i} - \frac{3}{2}\mathbf{j} = \left\langle 3, -\frac{3}{2}\right\rangle$



56. $\mathbf{v} = \frac{3}{4}\mathbf{w} = \frac{3}{4}(\mathbf{i} + 2\mathbf{j})$

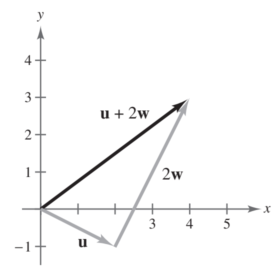
$= \frac{3}{4}\mathbf{i} + \frac{3}{2}\mathbf{j} = \left\langle \frac{3}{4}, \frac{3}{2}\right\rangle$



57. $\mathbf{v} = \mathbf{u} + 2\mathbf{w}$

$= (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j})$

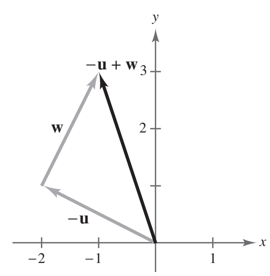
$= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3\rangle$



58. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$

$= -(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$

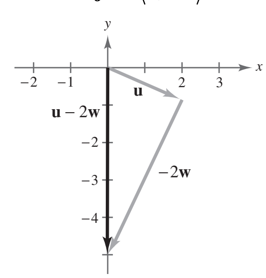
$= -\mathbf{i} + 3\mathbf{j} = \langle -1, 3\rangle$



59. $\mathbf{v} = \mathbf{u} - 2\mathbf{w}$

$= (2\mathbf{i} - \mathbf{j}) - 2(\mathbf{i} + 2\mathbf{j})$

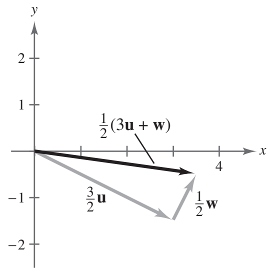
$= -5\mathbf{j} = \langle 0, -5\rangle$



60. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$

$= \frac{1}{2}(6\mathbf{i} - 3\mathbf{j} + \mathbf{i} + 2\mathbf{j})$

$= \frac{7}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} = \left\langle \frac{7}{2}, -\frac{1}{2} \right\rangle$



61. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$

$\tan\theta = \dfrac{-6}{6} = -1$

Since $\mathbf{v}$ lies in Quadrant IV, $\theta = 315°$.

62. $\mathbf{v} = -5\mathbf{i} + 4\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-5)^2 + 4^2} = \sqrt{41}$

$\tan\theta = -\frac{4}{5}$

Since $\mathbf{v}$ lies in Quadrant II, $\theta = 141.3°$.

63. $\mathbf{v} = 3(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

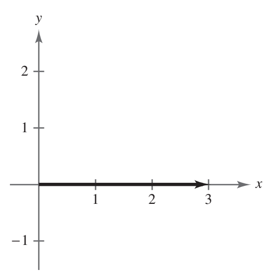
$\|\mathbf{v}\| = 3, \theta = 60°$

64. $\mathbf{v} = 8(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

$\|\mathbf{v}\| = 8, \theta = 135°$

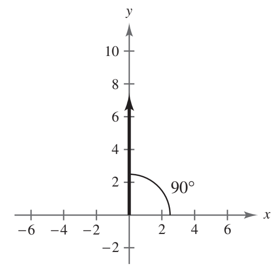
65. $\mathbf{v} = \langle 3\cos 0°, 3\sin 0° \rangle$

$= \langle 3, 0 \rangle$



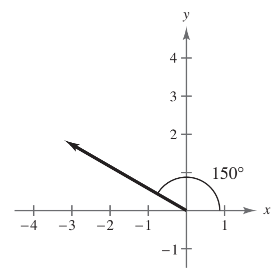
66. $\mathbf{v} = \left\langle 4\sqrt{3}\cos 90°, 4\sqrt{3}\sin 90° \right\rangle$

$= \left\langle 0, 4\sqrt{3} \right\rangle$



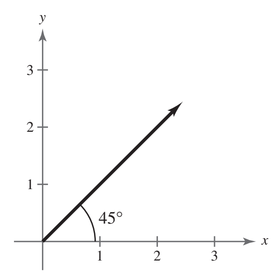
67. $\mathbf{v} = \left\langle \dfrac{7}{2}\cos 150°, \dfrac{7}{2}\sin 150° \right\rangle$

$= \left\langle -\dfrac{7\sqrt{3}}{4}, \dfrac{7}{4} \right\rangle$



68. $\mathbf{v} = \left\langle 2\sqrt{3}\cos 45°, 2\sqrt{3}\sin 45° \right\rangle$

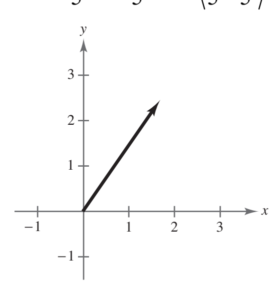
$= \left\langle \sqrt{6}, \sqrt{6} \right\rangle$



69. $\mathbf{v} = 3\left(\dfrac{1}{\sqrt{3^2 + 4^2}}\right)(3\mathbf{i} + 4\mathbf{j})$

$= \dfrac{3}{5}(3\mathbf{i} + 4\mathbf{j})$

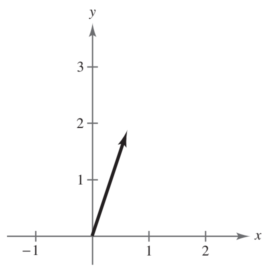
$= \dfrac{9}{5}\mathbf{i} + \dfrac{12}{5}\mathbf{j} = \left\langle \dfrac{9}{5}, \dfrac{12}{5} \right\rangle$



70. $\mathbf{v} = 2\left(\dfrac{1}{\sqrt{1^2 + 3^2}}\right)(\mathbf{i} + 3\mathbf{j})$

$= \dfrac{2}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j})$

$= \dfrac{\sqrt{10}}{5}\mathbf{i} + \dfrac{3\sqrt{10}}{5}\mathbf{j} = \left\langle \dfrac{\sqrt{10}}{5}, \dfrac{3\sqrt{10}}{5} \right\rangle$



71. $\mathbf{u} = \langle 4 \cos 60°, 4 \sin 60° \rangle = \langle 2, 2\sqrt{3} \rangle$

$\mathbf{v} = \langle 4 \cos 90°, 4 \sin 90° \rangle = \langle 0, 4 \rangle$

$\mathbf{u} + \mathbf{v} = \langle 2, 4 + 2\sqrt{3} \rangle$

72. $\mathbf{u} = \langle 20 \cos 45°, 20 \sin 45° \rangle = \langle 10\sqrt{2}, 10\sqrt{2} \rangle$

$\mathbf{v} = \langle 50 \cos 180°, 50 \sin 180° \rangle = \langle -50, 0 \rangle$

$\mathbf{u} + \mathbf{v} = \langle 10\sqrt{2} - 50, 10\sqrt{2} \rangle$

73. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

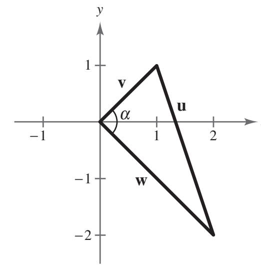
$\mathbf{w} = 2\mathbf{i} - 2\mathbf{j}$

$\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{2}$

$\|\mathbf{w}\| = 2\sqrt{2}$

$\|\mathbf{v} - \mathbf{w}\| = \sqrt{10}$



$\cos \alpha = \dfrac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\,\|\mathbf{w}\|} = \dfrac{2 + 8 - 10}{2\sqrt{2} \cdot 2\sqrt{2}} = 0$

$\alpha = 90°$

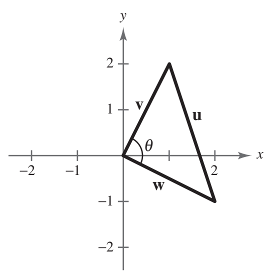
74. $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

$\mathbf{w} = 2\mathbf{i} - \mathbf{j}$

$\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$

$\cos \theta = \dfrac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\,\|\mathbf{w}\|} = \dfrac{5 + 5 - 10}{2\sqrt{5}\sqrt{5}} = 0$

$\theta = 90°$



75. Force One: $\mathbf{u} = 45\mathbf{i}$

Force Two: $\mathbf{v} = 60 \cos \theta\mathbf{i} + 60 \sin \theta\mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (45 + 60 \cos \theta)\mathbf{i} + 60 \sin \theta\mathbf{j}$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(45 + 60 \cos \theta)^2 + (60 \sin \theta)^2} = 90$$

$$2025 + 5400 \cos \theta + 3600 = 8100$$

$$5400 \cos \theta = 2475$$

$$\cos \theta = \tfrac{2475}{5400} \approx 0.4583$$

$$\theta \approx 62.7°$$

76. Force One: $\mathbf{u} = 3000\mathbf{i}$

Force Two: $\mathbf{v} = 1000 \cos \theta\mathbf{i} + 1000 \sin \theta\mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (3000 + 1000 \cos \theta)\mathbf{i} + 1000 \sin \theta\mathbf{j}$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(3000 + 1000 \cos \theta)^2 + (1000 \sin \theta)^2} = 3750$$

$$9{,}000{,}000 + 6{,}000{,}000 \cos \theta + 1{,}000{,}000 = 14{,}062{,}500$$

$$6{,}000{,}000 \cos \theta = 4{,}062{,}500$$

$$\cos \theta = \frac{4{,}062{,}500}{6{,}000{,}000} \approx 0.6771$$

$$\theta \approx 47.4°$$

77. Horizontal component of velocity: $1200 \cos 6° \approx 1193.4$ ft/sec

Vertical component of velocity: $1200 \sin 6° \approx 125.4$ ft/sec

78. Vertical component of velocity: $105 \sin(-3.5°) \approx -6.41$ mph

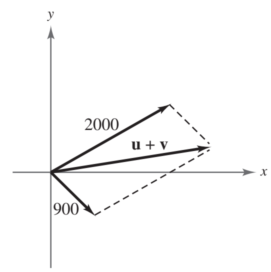
Horizontal component of velocity: $105 \cos(-3.5°) \approx 104.80$ mph

79.

$$\mathbf{u} = 300\mathbf{i}$$
$$\mathbf{v} = (125 \cos 45°)\mathbf{i} + (125 \sin 45°)\mathbf{j} = \frac{125}{\sqrt{2}}\mathbf{i} + \frac{125}{\sqrt{2}}\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} = \left(300 + \frac{125}{\sqrt{2}}\right)\mathbf{i} + \frac{125}{\sqrt{2}}\mathbf{j}$$
$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{\left(300 + \frac{125}{\sqrt{2}}\right)^2 + \left(\frac{125}{\sqrt{2}}\right)^2} \approx 398.32 \text{ newtons}$$
$$\tan\theta = \frac{\frac{125}{\sqrt{2}}}{300 + \left(\frac{125}{\sqrt{2}}\right)} \Rightarrow \theta \approx 12.8°$$

80.

$$\mathbf{u} = (2000 \cos 30°)\mathbf{i} + (2000 \sin 30°)\mathbf{j}$$
$$\approx 1732.05\mathbf{i} + 1000\mathbf{j}$$
$$\mathbf{v} = (900 \cos(-45°))\mathbf{i} + (900 \sin(-45°))\mathbf{j}$$
$$\approx 636.4\mathbf{i} + -636.4\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} \approx 2368.4\mathbf{i} + 363.6\mathbf{j}$$
$$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(2368.4)^2 + (363.6)^2} \approx 2396.2 \text{ newtons}$$
$$\tan\theta = \frac{363.6}{2368.4} \approx 0.1535 \Rightarrow \theta \approx 8.7°$$



81.

$$\mathbf{u} = (75 \cos 30°)\mathbf{i} + (75 \sin 30°)\mathbf{j} \approx 64.95\mathbf{i} + 37.5\mathbf{j}$$
$$\mathbf{v} = (100 \cos 45°)\mathbf{i} + (100 \sin 45°)\mathbf{j} \approx 70.71\mathbf{i} + 70.71\mathbf{j}$$
$$\mathbf{w} = (125 \cos 120°)\mathbf{i} + (125 \sin 120°)\mathbf{j} \approx -62.5\mathbf{i} + 108.3\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} + \mathbf{w} \approx 73.16\mathbf{i} + 216.5\mathbf{j}$$
$$\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \approx 228.5 \text{ pounds}$$
$$\tan\theta \approx \frac{216.5}{73.16} \approx 2.9593$$
$$\theta \approx 71.3°$$

82.

$$\mathbf{u} = (70 \cos 30°)\mathbf{i} - (70 \sin 30°)\mathbf{j} \approx 60.62\mathbf{i} - 35\mathbf{j}$$
$$\mathbf{v} = (40 \cos 45°)\mathbf{i} + (40 \sin 45°)\mathbf{j} \approx 28.28\mathbf{i} + 28.28\mathbf{j}$$
$$\mathbf{w} = (60 \cos 135°)\mathbf{i} + (60 \sin 135°)\mathbf{j} \approx -42.43\mathbf{i} + 42.43\mathbf{j}$$
$$\mathbf{u} + \mathbf{v} + \mathbf{w} = 46.48\mathbf{i} + 35.71\mathbf{j}$$
$$\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \approx 58.6 \text{ pounds}$$
$$\tan\theta \approx \frac{35.71}{46.47} \approx 0.7683$$
$$\theta \approx 37.5°$$

83. Left crane: $\mathbf{u} = \|\mathbf{u}\|(\cos 155.7°\mathbf{i} + \sin 155.7°\mathbf{j})$

Right crane: $\mathbf{v} = \|\mathbf{v}\|(\cos 44.5°\mathbf{i} + \sin 44.5°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = -20{,}240\mathbf{j}$

System of equations:

$\|\mathbf{u}\| \cos 155.7° + \|\mathbf{v}\| \cos 44.5° = 0$

$\|\mathbf{u}\| \sin 155.7° + \|\mathbf{v}\| \sin 44.5° = 20{,}240$

Solving this system of equations yields the following:

Left crane $= \|\mathbf{u}\| \approx 15{,}484$ pounds

Right crane $= \|\mathbf{v}\| \approx 19{,}786$ pounds

84. Left crane: $\mathbf{u} = \|\mathbf{u}\|(\cos 144.4°\mathbf{i} + \sin 144.4°\mathbf{j})$

Right crane: $\mathbf{v} = \|\mathbf{v}\|(\cos 40.4°\mathbf{i} + \sin 40.4°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = -20{,}240\mathbf{j}$

System of equations:

$\|\mathbf{u}\| \cos 144.4° + \|\mathbf{v}\| \cos 40.4° = 0$

$\|\mathbf{u}\| \sin 144.4° + \|\mathbf{v}\| \sin 40.4° = -20{,}240$

Solving this system of equations yields the following:

Left crane $= \|\mathbf{u}\| \approx 15{,}885$ pounds

Right crane $= \|\mathbf{v}\| \approx 16{,}961$ pounds

86. Cable $\overline{AC}$: $\mathbf{u} = 10\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant IV and its reference angle is $\arctan\left(\frac{12}{5}\right)$.

$$\mathbf{u} = \|\mathbf{u}\|\left[\cos\left(\arctan\frac{12}{5}\right)\mathbf{i} - \sin\left(\arctan\frac{12}{5}\right)\mathbf{j}\right]$$

Cable $\overline{BC}$: $\mathbf{v} = -20\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant III and its reference angle is $\arctan\left(\frac{6}{5}\right)$.

$$\mathbf{v} = \|\mathbf{v}\|\left[-\cos\left(\arctan\frac{6}{5}\right)\mathbf{i} - \sin\left(\arctan\frac{6}{5}\right)\mathbf{j}\right]$$

Resultant: $\mathbf{u} + \mathbf{v} = -5000\mathbf{j}$

$$\|\mathbf{u}\|\cos\left(\arctan\frac{12}{5}\right) - \|\mathbf{v}\|\cos\left(\arctan\frac{6}{5}\right) = 0$$

$$-\|\mathbf{u}\|\sin\left(\arctan\frac{12}{5}\right) - \|\mathbf{v}\|\sin\left(\arctan\frac{6}{5}\right) = -5000$$

Solving this system of equations yields:

$T_{AC} = \|\mathbf{u}\| \approx 3611.1$ pounds

$T_{BC} = \|\mathbf{v}\| \approx 2169.5$ pounds

85. Horizontal force: $\mathbf{u} = \|\mathbf{u}\|\mathbf{i}$

Weight: $\mathbf{w} = -\mathbf{j}$

Rope: $\mathbf{t} = \|\mathbf{t}\|(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

$$\mathbf{u} + \mathbf{w} + \mathbf{t} = \mathbf{0} \Rightarrow \begin{aligned} \|\mathbf{u}\| + \|\mathbf{t}\|\cos 135° &= 0 \\ -1 + \|\mathbf{t}\|\sin 135° &= 0 \end{aligned}$$

$\|\mathbf{t}\| \approx \sqrt{2}$ pounds

$\|\mathbf{u}\| \approx 1$ pound

87. Towline 1: $\mathbf{u} = \|\mathbf{u}\|(\cos 18°\mathbf{i} + \sin 18°\mathbf{j})$

Towline 2: $\mathbf{v} = \|\mathbf{u}\|(\cos 18°\mathbf{i} - \sin 18°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = 6000\mathbf{i}$

$$\|\mathbf{u}\| \cos 18° + \|\mathbf{u}\| \cos 18° = 6000$$
$$\|\mathbf{u}\| \approx 3154.4$$

So, the tension on each towline is $\|\mathbf{u}\| \approx 3154.4$ pounds.

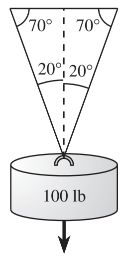
88. Rope 1: $\mathbf{u} = \|\mathbf{u}\|(\cos 70°\mathbf{i} - \sin 70°\mathbf{j})$

Rope 2: $\mathbf{v} = \|\mathbf{u}\|(-\cos 70°\mathbf{i} - \sin 70°\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = -100\mathbf{j}$

$$-\|\mathbf{u}\| \sin 70° - \|\mathbf{u}\| \sin 70° = -100$$
$$\|\mathbf{u}\| \approx 53.2$$

So, the tension of each rope is $\|\mathbf{u}\| \approx 53.2$ pounds.



89. $W = 100, \theta = 12°$

$$\sin \theta = \frac{F}{W}$$
$$F = W \sin \theta = 100 \sin 12° \approx 20.8 \text{ pounds}$$

90. $F = 600, \theta = 14°$

$$\sin \theta = \frac{F}{W}$$
$$W = \frac{F}{\sin \theta} = \frac{600}{\sin 14°} \approx 2480.1 \text{ pounds}$$

91. $F = 5000, W = 15{,}000$

$$\sin \theta = \frac{F}{W}$$
$$\sin \theta = \frac{5000}{15{,}000}$$
$$\theta = \sin^{-1} \frac{1}{3} \approx 19.5°$$

92. $W = 5000, \theta = 26°$

$$\sin \theta = \frac{F}{W}$$
$$F = W \sin \theta = 5000 \sin 26° \approx 2191.9 \text{ pounds}$$

93. Airspeed: $\mathbf{u} = (875 \cos 58°)\mathbf{i} - (875 \sin 58°)\mathbf{j}$

Groundspeed: $\mathbf{v} = (800 \cos 50°)\mathbf{i} - (800 \sin 50°)\mathbf{j}$

Wind: $\mathbf{w} = \mathbf{v} - \mathbf{u} = (800 \cos 50° - 875 \cos 58°)\mathbf{i} + (-800 \sin 50° + 875 \sin 58°)\mathbf{j}$

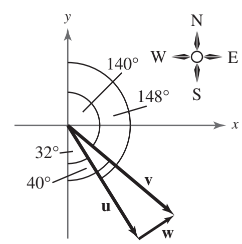
$$\approx 50.5507\mathbf{i} + 129.2065\mathbf{j}$$

Wind speed: $\|\mathbf{w}\| \approx \sqrt{(50.5507)^2 + (129.2065)^2} \approx 138.7$ kilometers per hour

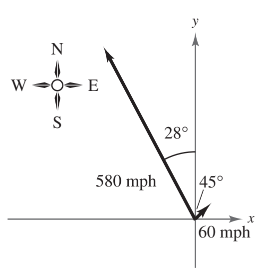
Wind direction: $\tan \theta \approx \dfrac{129.2065}{50.5507}$

$$\theta \approx 68.6°;\ 90° - \theta = 21.4°$$

Bearing: N 21.4° E



94. (a)



(b) The velocity vector $\mathbf{v}_w$ of the wind has a magnitude of 60 and a magnitude of 60 and a direction angle of 45°.

$$\mathbf{v}_w = \|\mathbf{v}_w\|(\cos \theta)\mathbf{i} + \|\mathbf{v}_w\|(\sin \theta)\mathbf{j}$$
$$= 60(\cos 45°)\mathbf{i} + 60(\sin 45°)\mathbf{j}$$
$$= 60[(\cos 45°)\mathbf{i} + (\sin 45°)\mathbf{j}]$$
$$= 60\langle \cos 45°, \sin 45° \rangle, \text{ or } \langle 30\sqrt{2}, 30\sqrt{2} \rangle$$

(c) The velocity vector $\mathbf{v}_j$ of the jet has a magnitude of 580 and a direction angle of 118°.

$$\mathbf{v}_j = \|\mathbf{v}_j\|(\cos \theta)\mathbf{i} + \|\mathbf{v}_j\|(\sin \theta)\mathbf{j}$$
$$= 580(\cos 118°)\mathbf{i} + 580(\sin 118°)\mathbf{j}$$
$$= 580[(\cos 118°)\mathbf{i} + (\sin 118°)\mathbf{j}]$$
$$= 580\langle \cos 118°, \sin 118° \rangle$$

(d) The velocity of the jet (in the wind) is

$$\begin{aligned}\mathbf{v} &= \mathbf{v}_w + \mathbf{v}_j \\ &= 60\langle \cos 45°, \sin 45° \rangle + 580\langle \cos 118°, \sin 118° \rangle \\ &= \langle 60 \cos 45° + 580 \cos 118°, 60 \sin 45° + 580 \sin 118° \rangle \\ &\approx \langle -229.87, 554.54 \rangle.\end{aligned}$$

The resultant speed of the jet is

$$\|\mathbf{v}\| = \sqrt{(-229.87)^2 + (554.54)^2} \approx 600.3 \text{ miles per hour.}$$

(e) If θ is the direction of the flight path, then

$$\tan \theta = \frac{554.54}{-229.87} \approx -2.4124.$$

Because θ lies in the Quadrant II, $\theta = 180° + \arctan(-2.4124) \approx 180° - 67.5° = 112.5°$.

The true bearing of the jet is $112.5° - 90° = 22.5°$ west of north, or $360° - 22.5° = 337.5°$.

95. True. Two directed line segments that have the same magnitude and direction are equivalent (see Example 1).

96. True. Given that $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$, then $\mathbf{v} = \mathbf{u}\|\mathbf{v}\|$.

97. True. If $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = 0$ is the zero vector, then $a = b = 0$. So, $a = -b$.

98. True. If $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ is a unit vector, then $\|\mathbf{u}\| = \sqrt{a^2 + b^2} = 1$ by the definition of the unit vector. So, $a^2 + b^2 = 1$.

99. The order of subtraction should be switched.

$$\mathbf{u} = \langle 6 - (-3), -1 - 4 \rangle = \langle 9, -5 \rangle$$

100. $\mathbf{v} = \langle -8, 5 \rangle$ and the $\mathbf{i}$ component is negative, so $\mathbf{v}$ lies in Quadrant II not Quadrant IV,

$$\theta' = \left| \arctan\left(-\frac{8}{5}\right) \right| \approx |-57.99°| = 57.99°$$

$$\theta = 180° - 57.99° = 122.01°$$

101. Let $\mathbf{v} = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$.

$$\|\mathbf{v}\| = \sqrt{\cos^2 \theta + \sin^2 \theta} = \sqrt{1} = 1$$

So, $\mathbf{v}$ is a unit vector for any value of θ.

102. The following program is written for a *T1-82, T1-83*, *T1-83 Plus* or *TI-84 Plus* graphing calculator. The program sketches two vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v} = c\mathbf{i} + d\mathbf{j}$ in standard position, and then sketches the vector difference $\mathbf{u} - \mathbf{v}$ using the parallelogram law.

```
PROGRAM: SUBVECT
:Input "ENTER A", A
:Input "ENTER B", B
:Input "ENTER C", C
:Input "ENTER D", D
:Line (0, 0, A, B)
:Line (0, 0, C, D)
:Pause
:A − C → E
:B − D → F
:Line (A, B, C, D)
:Line (A, B, E, F)
:Line (0, 0, E, F)
:Pause
:ClrDraw
:Stop
```

103. $\mathbf{u} = \langle 5 - 1, 2 - 6 \rangle = \langle 4, -4 \rangle$

$\mathbf{v} = \langle 9 - 4, 4 - 5 \rangle = \langle 5, -1 \rangle$

$\mathbf{u} - \mathbf{v} = \langle -1, -3 \rangle$ or $\mathbf{v} - \mathbf{u} = \langle 1, 3 \rangle$

104. $\mathbf{u} = \langle 80 - 10, 80 - 60 \rangle = \langle 70, 20 \rangle$

$\mathbf{v} = \langle -20 - (-100), 70 - 0 \rangle = \langle 80, 70 \rangle$

$\mathbf{u} - \mathbf{v} = \langle 70 - 80, 20 - 70 \rangle = \langle -10, -50 \rangle$

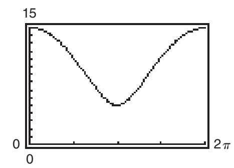
$\mathbf{v} - \mathbf{u} = \langle 80 - 70, 70 - 20 \rangle = \langle 10, 50 \rangle$

105. $\mathbf{F}_1 = \langle 10, 0 \rangle, \mathbf{F}_2 = 5\langle \cos\theta, \sin\theta \rangle$

(a) $\mathbf{F}_1 + \mathbf{F}_2 = \langle 10 + 5\cos\theta, 5\sin\theta \rangle$

$$\begin{aligned} \|\mathbf{F}_1 + \mathbf{F}_2\| &= \sqrt{(10 + 5\cos\theta)^2 + (5\sin\theta)^2} \\ &= \sqrt{100 + 100\cos\theta + 25\cos^2\theta + 25\sin^2\theta} \\ &= 5\sqrt{4 + 4\cos\theta + \cos^2\theta + \sin^2\theta} \\ &= 5\sqrt{4 + 4\cos\theta + 1} \\ &= 5\sqrt{5 + 4\cos\theta} \end{aligned}$$

(b)



(c) Range: $[5, 15]$

Maximum is 15 when $\theta = 0$.

Minimum is 5 when $\theta = \pi$.

(d) The magnitude of the resultant is never 0 because the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are not the same.

106. (a) True. **a** and **d** have the same magnitude, are parallel, and are pointing in opposite directions.

(b) True. **c** and **s** have the same magnitude, are parallel, and are pointing in the same direction.

(c) True. By definition of vector addition.

(d) False. $\mathbf{v} - \mathbf{w} = -\mathbf{s}$

(e) True.

$\mathbf{a} = -\mathbf{d}, \mathbf{w} = -\mathbf{d}, \mathbf{a} + \mathbf{w} = -\mathbf{d} + (-\mathbf{d}) = -2\mathbf{d}$

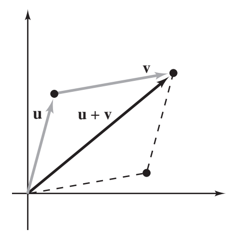
(f) True. $\mathbf{a} = -\mathbf{d}, \mathbf{a} + \mathbf{d} = -\mathbf{d} + \mathbf{d} = 0$

(g) False.

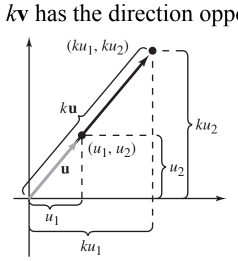
$\mathbf{u} - \mathbf{v} = 2\mathbf{u}$ and $-2(\mathbf{b} + \mathbf{t}) = -2(-2\mathbf{u}) = 4\mathbf{u}$

(h) True. $\mathbf{a} = \mathbf{w}, \mathbf{b} = \mathbf{t}, \mathbf{t} - \mathbf{w} = \mathbf{b} - \mathbf{a}$

107. (a) Answers will vary. *Sample answer:* To add two vectors **u** and **v** geometrically, first position them (without changing their lengths or directions) so that the initial point of the second vector **v** coincides with the terminal point of the first vector **u**. The sum $\mathbf{u} + \mathbf{v}$ is the vector formed by joining the initial point of the first vector **u** with the terminal point of the second vector **v**.



(b) Answers will vary. Sample Answer: Geometrically, the product of a vector **v** and a scalar k is the vector that is $|k|$ times as long as **v**. When k is positive, $k\mathbf{v}$ has the same direction as **v**, and when k is negative, $k\mathbf{v}$ has the direction opposite that of **v**.



108. (a) Vector. The velocity has both magnitude and direction.

(b) Scalar. The price has only magnitude.

(c) Scalar. The temperature has only magnitude.

(d) Vector. The weight has magnitude and direction

Section 3.4 Vectors and Dot Products

1. dot product

2. $u_1v_1 + u_2v_2$

3. $\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$

4. orthogonal

5. $\left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

6. $\left\|\text{proj}_{\overrightarrow{PQ}}\ \mathbf{F}\right\| \left\|\overrightarrow{PQ}\right\|$; $\mathbf{F} \cdot \overrightarrow{PQ}$

7. $\mathbf{u} = \langle 7, 1\rangle, \mathbf{v} = \langle -3, 2\rangle$

$\mathbf{u} \cdot \mathbf{v} = 7(-3) + 1(2) = -19$

8. $\mathbf{u} = \langle 6, 10\rangle, \mathbf{v} = \langle -2, 3\rangle$

$\mathbf{u} \cdot \mathbf{v} = 6(-2) + 10(3) = 18$

9. $\mathbf{u} = \langle -6, 2\rangle, \mathbf{v} = \langle 1, 3\rangle$

$\mathbf{u} \cdot \mathbf{v} = -6(1) + 2(3) = 0$

10. $\mathbf{u} = \langle -2, 5\rangle, \mathbf{v} = \langle -1, -8\rangle$

$\mathbf{u} \cdot \mathbf{v} = -2(-1) + 5(-8) = -38$

11. $\mathbf{u} = 4\mathbf{i} - 2\mathbf{j}, \mathbf{v} = \mathbf{i} - \mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 4(1) + (-2)(-1) = 6$

12. $\mathbf{u} = \mathbf{i} - 2\mathbf{j}, \mathbf{v} = -2\mathbf{i} - \mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 1(-2) + (-2)(-1) = 0$

13. $\mathbf{u} = \langle 3, 3\rangle$

$\mathbf{u} \cdot \mathbf{u} = 3(3) + 3(3) = 18$

The result is a scalar.

14. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle$

$3\mathbf{u} \cdot \mathbf{v} = 3\left[3(-4) + 3(2)\right] = 3(-6) = -18$

The result is a scalar.

15. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle$

$$\begin{aligned}(\mathbf{u} \cdot \mathbf{v})\mathbf{v} &= \left[3(-4) + 3(2)\right]\langle -4, 2\rangle \\ &= -6\langle -4, 2\rangle \\ &= \langle 24, -12\rangle\end{aligned}$$

The result is a vector.

16. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle, \mathbf{w} = \langle 3, -1\rangle$

$2\mathbf{v} = 2\langle -4, 2\rangle = \langle -8, 4\rangle$

$$\begin{aligned}(\mathbf{u} \cdot 2\mathbf{v})\mathbf{w} &= \left[3(-8) + 3(4)\right]\langle 3, -1\rangle \\ &= -12\langle 3, -1\rangle \\ &= \langle -36, 12\rangle\end{aligned}$$

The result is a vector.

17. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle, \mathbf{w} = \langle 3, -1\rangle$

$(\mathbf{v} \cdot \mathbf{0})\mathbf{w} = 0\langle 3, -1\rangle = \langle 0, 0\rangle = \mathbf{0}$

The result is a vector.

18. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle, \mathbf{w} = \langle 3, -1\rangle$

$$\begin{aligned}(\mathbf{u} + \mathbf{v}) \cdot \mathbf{0} &= \langle 3 + (-4), 3 + 2\rangle \cdot \langle 0, 0\rangle \\ &= \langle -1, 5\rangle \cdot \langle 0, 0\rangle \\ &= -1(0) + 5(0) \\ &= 0\end{aligned}$$

The result is a scalar.

19. $\mathbf{w} = \langle 3, -1\rangle$

$\|\mathbf{w}\| - 1 = \sqrt{3^2 + (-1)^2} - 1 = \sqrt{10} - 1$

The result is a scalar.

20. $\mathbf{u} = \langle 3, 3\rangle$

$2 - \|\mathbf{u}\| = 2 - \sqrt{3^2 + 3^2} = 2 - \sqrt{18} = 2 - 3\sqrt{2}$

The result is a scalar.

21. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle, \mathbf{w} = \langle 3, -1\rangle$

$$\begin{aligned}(\mathbf{u} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{w}) &= \left[3(-4) + 3(2)\right] - \left[3(3) + 3(-1)\right] \\ &= -6 - 6 \\ &= -12\end{aligned}$$

The result is a scalar.

22. $\mathbf{u} = \langle 3, 3\rangle, \mathbf{v} = \langle -4, 2\rangle, \mathbf{w} = \langle 3, -1\rangle$

$$\begin{aligned}(\mathbf{v} \cdot \mathbf{u}) - (\mathbf{w} \cdot \mathbf{v}) &= \left[-4(3) + 2(3)\right] - \left[3(-4) + (-1)(2)\right] \\ &= -6 - (-14) \\ &= 8\end{aligned}$$

The result is a scalar.

23. $\mathbf{u} = \langle -8, 15\rangle$

$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(-8)(-8) + 15(15)} = \sqrt{289} = 17$

24. $\mathbf{u} = \langle 4, -6 \rangle$

$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{4(4) + (-6)(-6)} = \sqrt{52} = 2\sqrt{13}$

25. $\mathbf{u} = 20\mathbf{i} + 25\mathbf{j}$

$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(20)^2 + (25)^2} = \sqrt{1025} = 5\sqrt{41}$

26. $\mathbf{u} = 12\mathbf{i} - 16\mathbf{j}$

$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{12(12) + (-16)(-16)}$

$= \sqrt{400} = 20$

27. $\mathbf{u} = 6\mathbf{j}$

$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(0)^2 + (6)^2} = \sqrt{36} = 6$

28. $\mathbf{u} = -21\mathbf{i}$

$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{(-21)(-21) + 0(0)}$

$= \sqrt{21^2} = 21$

29. $\mathbf{u} = \langle 1, 0 \rangle, \mathbf{v} = \langle 0, -2 \rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{0}{(1)(2)} = 0$$

$$\theta = \frac{\pi}{2} \text{ radians}$$

30. $\mathbf{u} = \langle 3, 2 \rangle, \mathbf{v} = \langle 4, 0 \rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{3(4) + 2(0)}{\sqrt{13}(4)}$$

$$= \frac{3}{\sqrt{13}} \approx 0.83205$$

$$\theta \approx 0.59 \text{ radian}$$

31. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = -2\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = -\frac{8}{(5)(2)}$$

$$\theta = \arccos\left(-\frac{4}{5}\right)$$

$$\theta \approx 2.50 \text{ radians}$$

32. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

$$= \frac{2(1) + (-3)(-2)}{\sqrt{2^2 + 3^2}\sqrt{1^2 + 2^2}}$$

$$= \frac{8}{\sqrt{65}} \approx 0.992278$$

$$\theta \approx 0.12 \text{ radian}$$

33. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = 6\mathbf{i} - 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

$$= \frac{2(6) + (-1)(-3)}{\sqrt{2^2 + (-1)^2}\sqrt{6^2 + (-3)^2}}$$

$$= \frac{15}{\sqrt{225}} = 1$$

$$\theta = 0$$

34. $\mathbf{u} = 5\mathbf{i} + 5\mathbf{j}, \mathbf{v} = -6\mathbf{i} + 6\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = 0$$

$$\theta = \frac{\pi}{2}$$

35. $\mathbf{u} = -6\mathbf{i} - 3\mathbf{j}, \mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$

$$\cos\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-6(-8) + (-3)(4)}{\sqrt{45}\sqrt{80}} = \frac{36}{60} = 0.6$$

$$\theta \approx 0.93 \text{ radian}$$

36. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{2(4) + (-3)(3)}{\sqrt{13}\sqrt{25}} \approx -0.0555$$

$$\theta \approx 1.63 \text{ radians}$$

37. $\mathbf{u} = \left(\cos\frac{\pi}{3}\right)\mathbf{i} + \left(\sin\frac{\pi}{3}\right)\mathbf{j} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$

$$\mathbf{v} = \left(\cos\frac{3\pi}{4}\right)\mathbf{i} + \left(\sin\frac{3\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = 1$$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \mathbf{u} \cdot \mathbf{v}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2} + \sqrt{6}}{4}$$

$$\theta = \arccos\left(\frac{-\sqrt{2} + \sqrt{6}}{4}\right) = \frac{5\pi}{12}$$

38. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

$\mathbf{v} = \cos\left(\frac{5\pi}{4}\right)\mathbf{i} + \sin\left(\frac{5\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2}\right)}{1.1} = \frac{-1}{1} = -1$$

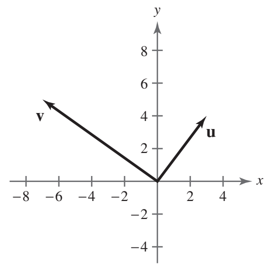
$\theta = \pi$

39. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$

$\mathbf{v} = -7\mathbf{i} + 5\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{3(-7) + 4(5)}{3\sqrt{74}} = \frac{-1}{5\sqrt{74}} \approx -0.0232$$

$\theta \approx 91.33°$



40. $\mathbf{u} = 6\mathbf{i} - 3\mathbf{j}$

$\mathbf{v} = -4\mathbf{i} - 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{6(-4) + (-3)(-4)}{12\sqrt{10}} = \frac{-12}{12\sqrt{10}} = -\frac{1}{\sqrt{10}}$$

$$\cos^{-1}\left(\frac{-1}{\sqrt{10}}\right) = \theta \Rightarrow \theta \approx 108.43°$$

41. $\mathbf{u} = -5\mathbf{i} - 5\mathbf{j}$

$\mathbf{v} = -8\mathbf{i} + 8\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-5(-8) + (-5)(8)}{\sqrt{50}\sqrt{128}} = 0$$

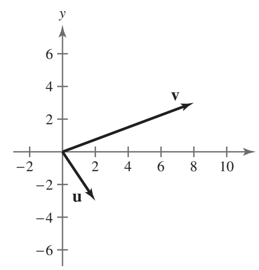
$\theta = 90°$

42. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$

$\mathbf{v} = 8\mathbf{i} + 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{2(8) + (-3)(3)}{\sqrt{13}\sqrt{73}} = \frac{7}{\sqrt{13}\sqrt{73}}$$

$$\cos^{-1}\left[\frac{7}{\sqrt{13}\sqrt{73}}\right] = 0 \Rightarrow \theta \approx 76.87°$$



43. $P = (1, 2), Q = (3, 4), R = (2, 5)$

$\overrightarrow{PQ} = \langle 2, 2\rangle, \overrightarrow{PR} = \langle 1, 3\rangle, \overrightarrow{QR} = \langle -1, 1\rangle$

$$\cos\alpha = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\|\overrightarrow{PQ}\|\|\overrightarrow{PR}\|} = \frac{8}{(2\sqrt{2})(\sqrt{10})} \Rightarrow \alpha = \arccos\frac{2}{\sqrt{5}} \approx 26.57°$$

$$\cos\beta = \frac{\overrightarrow{PQ}\cdot\overrightarrow{QR}}{\|\overrightarrow{PQ}\|\|\overrightarrow{QR}\|} = 0 \Rightarrow \beta = 90°$$

$\gamma = 180° - 26.57° - 90° = 63.43°$

44. $P = (-3, -4), Q = (1, 7), R = (8, 2)$

$\overline{PQ} = \langle 4, 11\rangle, \overline{QR} = \langle 7, -5\rangle,$

$\overline{PR} = \langle 11, 6\rangle, \overline{QP} = \langle -4, -11\rangle$

$$\cos\alpha = \frac{\overline{PQ}\cdot\overline{PR}}{\|\overline{PQ}\|\|\overline{PR}\|} = \frac{110}{(\sqrt{137})(\sqrt{157})} \Rightarrow \alpha \approx 41.41°$$

$$\cos\beta = \frac{\overline{QR}\cdot\overline{QP}}{\|\overline{QR}\|\|\overline{QP}\|} = \frac{27}{(\sqrt{74})(\sqrt{137})} \Rightarrow \beta \approx 74.45°$$

$$\gamma \approx 180° - 41.41° - 74.45° = 64.14°$$

45. $P = (-3, 0), Q = (2, 2), R = (0, 6)$

$\overline{QP} = \langle -5, -2\rangle, \overline{PR} = \langle 3, 6\rangle, \overline{QR} = \langle -2, 4\rangle, \overline{PQ} = \langle 5, 2\rangle$

$$\cos\alpha = \frac{\overline{PQ}\cdot\overline{PR}}{\|\overline{PQ}\|\|\overline{PR}\|} = \frac{27}{\sqrt{29}\sqrt{45}} \Rightarrow \alpha \approx 41.63°$$

$$\cos\beta = \frac{\overline{QP}\cdot\overline{QR}}{\|\overline{QP}\|\|\overline{PR}\|} = \frac{2}{\sqrt{29}\sqrt{20}} \Rightarrow \beta \approx 85.24°$$

$$\delta = 180° - 41.63° - 85.24° = 53.13°$$

46. $P = (-3, 5), Q = (-1, 9), R = (7, 9)$

$\overline{PQ} = \langle 2, 4\rangle, \overline{QR} = \langle 8, 0\rangle,$

$\overline{PR} = \langle 10, 4\rangle, \overline{QP} = \langle -2, -4\rangle$

$$\cos\alpha = \frac{\overline{PQ}\cdot\overline{PR}}{\|\overline{PQ}\|\|\overline{PR}\|} = \frac{36}{(\sqrt{20})(\sqrt{116})} \Rightarrow \alpha \approx 41.63°$$

$$\cos\beta = \frac{\overline{QR}\cdot\overline{QP}}{\|\overline{QR}\|\|\overline{QP}\|} = \frac{-16}{8(\sqrt{20})} \Rightarrow \beta \approx 116.57°$$

$$\gamma \approx 180° - 41.6° - 116.6° = 21.80°$$

47. $\|\mathbf{u}\| = 4, \|\mathbf{v}\| = 10, \theta = \frac{2\pi}{3}$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta\\ &= (4)(10)\cos\frac{2\pi}{3}\\ &= 40\left(-\frac{1}{2}\right)\\ &= -20\end{aligned}$$

48. $\|\mathbf{u}\| = 4$

$\|\mathbf{v}\| = 12$

$\theta = \frac{\pi}{3}$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta\\ &= (4)(12)\cos\frac{\pi}{3}\\ &= (4)(12)\left(\frac{1}{2}\right) = 24\end{aligned}$$

49. $\|\mathbf{u}\| = 100, \|\mathbf{v}\| = 250, \theta = \frac{\pi}{6}$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta\\ &= (100)(250)\cos\frac{\pi}{6}\\ &= 25{,}000\cdot\frac{\sqrt{3}}{2}\\ &= 12{,}500\sqrt{3}\end{aligned}$$

50. $\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 36, \theta = \frac{3\pi}{4}$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta\\ &= (9)(36)\cos\frac{3\pi}{4}\\ &= 324\left(-\frac{\sqrt{2}}{2}\right)\\ &= -162\sqrt{2} \approx -229.1\end{aligned}$$

51. $\mathbf{u} = \langle 3, 15\rangle, \mathbf{v} = \langle -1, 5\rangle$

$\mathbf{u} \neq k\mathbf{v} \Rightarrow$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \Rightarrow$ Not orthogonal

Neither

52. $\mathbf{u} = \langle 30, 12\rangle, \mathbf{v} = \left\langle \frac{1}{2}, -\frac{5}{4}\right\rangle$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= 30\left(\frac{1}{2}\right) + 12\left(-\frac{5}{4}\right)\\ &= 15 - 15 = 0\end{aligned}$$

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

53. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j}, \mathbf{v} = -\mathbf{i} - \mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

54. $\mathbf{u} = \frac{1}{4}(3\mathbf{i} - \mathbf{j}), \mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \Rightarrow$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \Rightarrow$ Not orthogonal

Neither

55. $\mathbf{u} = 1, \mathbf{v} = -2\mathbf{i} + 2\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \Rightarrow$ Not parallel

$\mathbf{u} \cdot \mathbf{v} \neq 0 \Rightarrow$ Not orthogonal

Neither

56. $\mathbf{u} = \langle \cos\theta, \sin\theta \rangle$

$\mathbf{v} = \langle \sin\theta, -\cos\theta \rangle$

$\mathbf{u} \cdot \mathbf{v} = 0 \Rightarrow$ **u** and **v** are orthogonal.

57. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle 6, 1 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{14}{37}\langle 6, 1 \rangle = \frac{1}{37}\langle 84, 14 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 2 \rangle - \frac{14}{37}\langle 6, 1 \rangle = \left\langle -\frac{10}{37}, \frac{60}{37} \right\rangle = \frac{10}{37}\langle -1, 6 \rangle = \frac{1}{37}\langle -10, 60 \rangle$$

$$\mathbf{u} = \frac{1}{37}\langle 84, 14 \rangle + \frac{1}{37}\langle -10, 60 \rangle = \langle 2, 2 \rangle$$

58. $\mathbf{u} = \langle 0, 3 \rangle, \mathbf{v} = \langle 2, 15 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{45}{229}\langle 2, 15 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3 \rangle - \frac{45}{229}\langle 2, 15 \rangle = \left\langle -\frac{90}{229}, \frac{12}{229} \right\rangle$$

$$= \frac{6}{229}\langle -15, 2 \rangle$$

$$\mathbf{u} = \frac{45}{229}\langle 2, 15 \rangle + \frac{6}{229}\langle -15, 2 \rangle = \langle 0, 3 \rangle$$

59. $\mathbf{u} = \langle 4, 2 \rangle, \mathbf{v} = \langle 1, -2 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = 0\langle 1, -2 \rangle = \langle 0, 0 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2 \rangle - \langle 0, 0 \rangle = \langle 4, 2 \rangle$$

$$\mathbf{u} = \langle 4, 2 \rangle + \langle 0, 0 \rangle = \langle 4, 2 \rangle$$

60. $\mathbf{u} = \langle -3, -2 \rangle, \mathbf{v} = \langle -4, -1 \rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{14}{17}\right)\langle -4, -1 \rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -3, -2 \rangle - \frac{14}{17}\langle -4, -1 \rangle = \frac{5}{17}\langle 1, -4 \rangle$$

$$\mathbf{u} = \frac{14}{17}\langle -4, -1 \rangle + \frac{5}{17}\langle -1, -4 \rangle = \langle -3, -2 \rangle$$

61. $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u}$ because **u** and **v** are parallel.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{3(6) + 2(4)}{\left(\sqrt{6^2 + 4^2}\right)^2}\langle 6, 4 \rangle = \frac{1}{2}\langle 6, 4 \rangle = \langle 3, 2 \rangle = \mathbf{u}$$

62. $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u}$ because **u** and **v** are parallel.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{-3(6) + (-2)(4)}{\left(\sqrt{6^2 + 4^2}\right)^2}\langle 6, 4 \rangle = -\frac{1}{2}\langle 6, 4 \rangle = \langle -3, -2 \rangle = \mathbf{u}$$

63. Because **u** and **v** are orthogonal, $\mathbf{u} \cdot \mathbf{v} = 0$ and $\text{proj}_{\mathbf{v}}\mathbf{u} = 0$.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = 0, \text{ because } \mathbf{u} \cdot \mathbf{v} = 0.$$

64. Because **u** and **v** are orthogonal, the projection of **u** onto **v** is 0.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = 0, \text{ because } \mathbf{u} \cdot \mathbf{v} = 0.$$

65. $\mathbf{u} = \langle 3, 5 \rangle$

For **v** to be orthogonal to **u**, $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\langle -5, 3 \rangle$ and $\langle 5, -3 \rangle$

66. $\mathbf{u} = \langle -8, 3\rangle$

For $\mathbf{v}$ to be orthogonal to $\mathbf{u}$, $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\langle 3, 8\rangle$, $\langle -3, -8\rangle$

67. $\mathbf{u} = \frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j}$

For $\mathbf{u}$ and $\mathbf{v}$ to be orthogonal, $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\mathbf{v} = \frac{2}{3}\mathbf{i} + \frac{1}{2}\mathbf{j}$ and $\mathbf{v} = -\frac{2}{3}\mathbf{i} - \frac{1}{2}\mathbf{j}$

68. $\mathbf{u} = -\frac{5}{2}\mathbf{i} - 3\mathbf{j}$

For $\mathbf{v}$ to be orthogonal to $\mathbf{u}$, $\mathbf{u} \cdot \mathbf{v}$ must equal 0.

Two possibilities: $\mathbf{v} = 3\mathbf{i} - \frac{5}{2}\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} + \frac{5}{2}\mathbf{j}$

69. Work $= \left\|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\right\| \left\|\overrightarrow{PQ}\right\|$ where $\overrightarrow{PQ} = \langle 4, 7\rangle$ and $\mathbf{v} = \langle 1, 4\rangle$.

$$\text{proj}_{\overrightarrow{PQ}}\mathbf{v} = \left(\frac{\mathbf{v} \cdot \overrightarrow{PQ}}{\left\|\overrightarrow{PQ}\right\|^2}\right)\overrightarrow{PQ} = \left(\frac{32}{65}\right)\langle 4, 7\rangle$$

$$\text{Work} = \left\|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\right\| \left\|\overrightarrow{PQ}\right\| = \left(\frac{32\sqrt{65}}{65}\right)\left(\sqrt{65}\right) = 32$$

70. $P = (1, 3), Q = (-3, 5), \mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$

$$\begin{aligned}\text{Work} &= \mathbf{v} \cdot \overrightarrow{PQ}\\ &= (-2\mathbf{i} + 3\mathbf{j}) \cdot (-4\mathbf{i} + 2\mathbf{j})\\ &= (-2)(-4) + 3(2) = 14\end{aligned}$$

71. (a) $\mathbf{u} \cdot \mathbf{v} = 1225(12.20) + 2445(8.50)$
$= 35{,}727.5$

The total amount paid to the employees is \$35,727.50.

(b) To increase wages by 2%, use scalar multiplication to multiply 1.02 by $\mathbf{v}$.

72. $\mathbf{u} = \langle 3140, 2750\rangle, \mathbf{v} = \langle 2.25, 1.75\rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = 3140(2.25) + 2750(1.75) = 11{,}877.5$

The total revenue earned by selling the hot dogs and hamburgers is \$11,877.50.

(b) Increase prices by 2.5%: $1.025\mathbf{v}$

The operation is scalar multiplication.

73. (a) Force due to gravity:

$\mathbf{F} = -30{,}000\mathbf{j}$

Unit vector along hill:

$\mathbf{v} = (\cos d)\mathbf{i} + (\sin d)\mathbf{j}$

Projection of $\mathbf{F}$ onto $\mathbf{v}$:

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{F} = \left(\frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = (\mathbf{F} \cdot \mathbf{v})\mathbf{v} = -30{,}000 \sin d\mathbf{v}$$

The magnitude of the force is $30{,}000 \sin d$.

(b)

d	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Force	0	523.6	1047.0	1570.1	2092.7	2614.7	3135.9	3656.1	4175.2	4693.0	5209.4

(c) Force perpendicular to the hill when $d = 5°$:

$$\text{Force} = \sqrt{(30{,}000)^2 - (2614.7)^2} \approx 29{,}885.8 \text{ pounds}$$

74. Force due to gravity: $\mathbf{F} = -5400\mathbf{j}$

Unit vector along hill: $\mathbf{v} = (\cos 10°)\mathbf{i} + (\sin 10°)\mathbf{j}$

Projection of $\mathbf{F}$ onto $\mathbf{v}$:
$$\begin{aligned}\mathbf{w}_1 &= \text{proj}_{\mathbf{v}}\mathbf{F}\\ &= \left(\frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}\\ &= (\mathbf{F}\cdot\mathbf{v})\mathbf{v} \text{ because } \mathbf{v} \text{ is a unit vector, } \|\mathbf{v}\| = 1\\ &= \left[(0)(\cos 10°) + (-5400)(\sin 10°)\right]\mathbf{v}\\ &= -5400(\sin 10°)\mathbf{v} = -937.7\mathbf{v}\end{aligned}$$

The magnitude of the force is 937.7, so a force of 937.7 pounds is required to keep the vehicle from rolling down the hill.

Force perpendicular to the hill: $\text{Force} = \sqrt{(5400)^2 - (937)^2} \approx 5318.0$ pounds

75. $\text{Work} = (245)(3) = 735$ newton-meters

76. $\text{Work} = (2400)(5) = 12{,}000$ foot-pounds

77. $\text{Work} = (\cos 30°)(45)(20) \approx 779.4$ foot-pounds

78. $\text{Work} = (\cos 25°)(50)(15) \approx 679.7$ foot-pounds

79. $\text{Work} = (\cos 35°)(15{,}691)(800)$
$\approx 10{,}282{,}651.78$ newton-meters

80. $\text{Work} = (\cos 30°)(250)(100) \approx 21{,}650.64$ foot-pounds

81.
$$\begin{aligned}\text{Work} &= (\cos\theta)\|\mathbf{F}\|\,\|\overrightarrow{PQ}\|\\ &= (\cos 20°)(25 \text{ pounds})(50 \text{ feet})\\ &\approx 1174.62 \text{ foot-pounds}\end{aligned}$$

82. $\text{Work} = (\cos 22°)(35)(200) \approx 6490.3$ foot-pounds

83. False. Work is represented by a scalar.

84. True.

$W = \mathbf{F}\cdot\overrightarrow{PQ} = 0$ when $\mathbf{F}$ and $\overrightarrow{PQ}$ are orthogonal $(\cos 90° = 0)$.

85. A dot product is a scalar, not a vector.

$\mathbf{v}\cdot\mathbf{0} = \langle -3, 5\rangle \cdot \langle 0, 0\rangle = (-3)(0) + (5)(0) = 0$

86. The dot product is the sum of the two products, not the difference.
$$\begin{aligned}\mathbf{u}\cdot 2\mathbf{v} &= \langle 2, -1\rangle \cdot 2\langle -3, 5\rangle\\ &= \langle 2, -1\rangle \cdot \langle -6, 10\rangle\\ &= (2)(-6) + (-1)(10)\\ &= -12 - 10 = -22\end{aligned}$$

87. $\mathbf{u}\cdot\mathbf{v} = \langle 8, 4\rangle \cdot \langle 2, -k\rangle = 16 - 4k = 0$
$$\begin{aligned}16 - 4k &= 0\\ -4k &= -16\\ k &= 4\end{aligned}$$

88. $\mathbf{u}\cdot\mathbf{v} = \langle -3k, 5\rangle \cdot \langle 2, -4\rangle = -6k - 20 = 0$
$$\begin{aligned}-6k - 20 &= 0\\ -6k &= 20\\ k &= -\frac{10}{3}\end{aligned}$$

89.
$$\begin{aligned}\mathbf{u}\cdot\mathbf{u} &= \|\mathbf{u}\|^2\\ &= 1^2 = 1\end{aligned}$$

90. (a) $\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal and $\theta = \dfrac{\pi}{2}$.

(b) $\mathbf{u}\cdot\mathbf{v} > 0 \Rightarrow \cos\theta > 0 \Rightarrow 0 \le \theta < \dfrac{\pi}{2}$

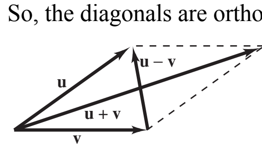
(c) $\mathbf{u}\cdot\mathbf{v} < 0 \Rightarrow \cos\theta < 0 \Rightarrow \dfrac{\pi}{2} < \theta \le \pi$

91. (a) $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are parallel.

(b) $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{0} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

92. In a rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.
$$\begin{aligned}(\mathbf{u} + \mathbf{v})\cdot(\mathbf{u} - \mathbf{v}) &= (\mathbf{u} + \mathbf{v})\cdot\mathbf{u} - (\mathbf{u} + \mathbf{v})\cdot\mathbf{v}\\ &= \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{v}\\ &= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0\end{aligned}$$

So, the diagonals are orthogonal.



93. Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$.

$$\mathbf{u} - \mathbf{v} = \langle u_1 - v_1, u_2 - v_2 \rangle$$

$$\begin{aligned} \|\mathbf{u} - \mathbf{v}\|^2 &= (u_1 - v_1)^2 + (u_2 - v_2)^2 \\ &= u_1^2 - 2u_1v_1 + v_1^2 + u_2^2 - 2u_2v_2 + v_2^2 \\ &= u_1^2 + u_2^2 + v_1^2 + v_2^2 - 2u_1v_1 - 2u_2v_2 \\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(u_1v_1 + u_2v_2) \\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v} \end{aligned}$$

Review Exercises for Chapter 3

1. Given: $A = 38°, B = 70°, a = 8$

$C = 180° - 38° - 70° = 72°$

$b = \dfrac{a \sin B}{\sin A} = \dfrac{8 \sin 70°}{\sin 38°} \approx 12.21$

$c = \dfrac{a \sin C}{\sin A} = \dfrac{8 \sin 72°}{\sin 38°} \approx 12.36$

2. Given: $A = 22°, B = 121°, a = 19$

$C = 180° - 22° - 121° = 37°$

$b = \dfrac{a \sin B}{\sin A} = \dfrac{19 \sin 121°}{\sin 22°} \approx 43.48$

$c = \dfrac{a \sin C}{\sin A} = \dfrac{19 \sin 37°}{\sin 22°} \approx 30.52$

3. Given: $B = 72°, C = 82°, b = 54$

$A = 180° - 72° - 82° = 26°$

$a = \dfrac{b \sin A}{\sin B} = \dfrac{54 \sin 26°}{\sin 72°} \approx 24.89$

$c = \dfrac{b \sin C}{\sin B} = \dfrac{54 \sin 82°}{\sin 72°} \approx 56.23$

4. Given: $B = 10°, C = 20°, c = 33$

$A = 180° - B - C = 150°$

$a = \dfrac{c \sin A}{\sin C} = \dfrac{33 \sin 150°}{\sin 20°} \approx 48.24$

$b = \dfrac{c \sin B}{\sin C} = \dfrac{33 \sin 10°}{\sin 20°} \approx 16.75$

5. Given: $A = 16°, B = 98°, c = 8.4$

$C = 180° - 16° - 98° = 66°$

$a = \dfrac{c \sin A}{\sin C} = \dfrac{8.4 \sin 16°}{\sin 66°} \approx 2.53$

$b = \dfrac{c \sin B}{\sin C} = \dfrac{8.4 \sin 98°}{\sin 66°} \approx 9.11$

6. Given: $A = 95°, B = 45°, c = 104.8$

$C = 180° - A - B = 40°$

$a = \dfrac{c \sin A}{\sin C} = \dfrac{104.8 \sin 95°}{\sin 40°} \approx 162.42$

$b = \dfrac{c \sin B}{\sin C} = \dfrac{104.8 \sin 45°}{\sin 40°} \approx 115.29$

7. Given: $A = 24°, C = 48°, b = 27.5$

$B = 180° - 24° - 48° = 108°$

$a = \dfrac{b \sin A}{\sin B} = \dfrac{27.5 \sin 24°}{\sin 108°} \approx 11.76$

$c = \dfrac{b \sin C}{\sin B} = \dfrac{27.5 \sin 48°}{\sin 108°} \approx 21.49$

8. Given: $B = 64°, C = 36°, a = 367$

$A = 180° - B - C = 80°$

$b = \dfrac{a \sin B}{\sin A} = \dfrac{367 \sin 64°}{\sin 80°} \approx 334.95$

$c = \dfrac{a \sin C}{\sin A} = \dfrac{367 \sin 36°}{\sin 80°} \approx 219.04$

9. Given: $B = 150°, b = 30, c = 10$

$\sin C = \dfrac{c \sin B}{b} = \dfrac{10 \sin 150°}{30} \approx 0.1667 \Rightarrow C \approx 9.59°$

$A \approx 180° - 150° - 9.59° = 20.41°$

$a = \dfrac{b \sin A}{\sin B} = \dfrac{30 \sin 20.41°}{\sin 150°} \approx 20.92$

10. Given: $B = 150°, a = 10, b = 3$

$$\sin A = \frac{a \sin B}{b} = \frac{10 \sin 150°}{3} \approx 1.67 > 1$$

No solution

11. $A = 75°, a = 51.2, b = 33.7$

$$\sin B = \frac{b \sin A}{a} = \frac{33.7 \sin 75°}{51.2} \approx 0.6358 \Rightarrow B \approx 39.48°$$

$$C \approx 180° - 75° - 39.48° = 65.52°$$

$$c = \frac{a \sin C}{\sin A} = \frac{51.2 \sin 65.52°}{\sin 75°} \approx 48.24$$

12. Given: $B = 25°, a = 6.2, b = 4$

$$\sin A = \frac{a \sin B}{b} \approx 0.65506 \Rightarrow A \approx 40.92° \text{ or } 139.08°$$

Case 1: $A \approx 40.92°$
$C \approx 180° - 25° - 40.92° = 114.08°$
$c \approx 8.64$

Case 2: $A \approx 139.08°$
$C \approx 180° - 25° - 139.08° = 15.92°$
$c \approx 2.60$

13. $A = 33°, b = 7, c = 10$

$$\text{Area} = \tfrac{1}{2}bc \sin A = \tfrac{1}{2}(7)(10) \sin 33° \approx 19.06$$

14. $B = 80°, a = 4, c = 8$

$$\text{Area} = \tfrac{1}{2}ac \sin B = \tfrac{1}{2}(4)(8)(0.9848) \approx 15.8$$

15. $C = 119°, a = 18, b = 6$

$$\text{Area} = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}(18)(6) \sin 119° \approx 47.23$$

16. $A = 11°, b = 22, c = 21$

$$\text{Area} = \tfrac{1}{2}bc \sin A \approx \tfrac{1}{2}(22)(21)(0.1908) \approx 44.1$$

17. $\text{Area} = \dfrac{1}{2}ac \sin B = \dfrac{1}{2}(105)(64) \sin(72° 30') \approx 3204.5$

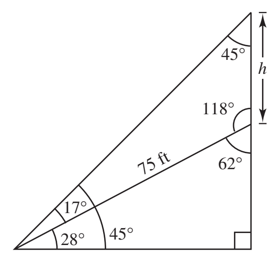
18. $C = 84° 30', a = 16, b = 20$

$$\text{Area} = \frac{1}{2}ab \sin C$$
$$= \left(\frac{1}{2}\right)(16)(20) \sin 84.5° \approx 159.3$$

19. $\dfrac{h}{\sin 17°} = \dfrac{75}{\sin 45°}$

$$h = \frac{75 \sin 17°}{\sin 45°}$$

$h \approx 31.01$ feet



20. The triangle of base 400 feet formed by the two angles of sight to the tree has base angles of $90° - 22° 30' = 67° 30'$, or $67.5°$, and $90° - 15° = 75°$.

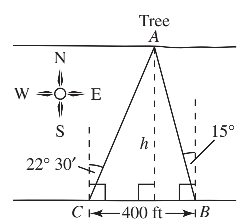
The angle at the tree measures $180° - 67.5° - 75° = 37.5°$.

$$b = \frac{400 \sin 75°}{\sin 37.5°} \approx 634.683$$

$$h = 634.683 \sin 67.5°$$

$$h \approx 586.4$$

The width of the river is about 586.4 feet.



21. Given: $a = 8, b = 14, c = 17$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{64 + 196 - 289}{2(8)(14)} \approx -0.1295 \Rightarrow C \approx 97.44°$$

$$\sin B = \frac{b \sin C}{c} \approx \frac{14 \sin 97.44}{17} \approx 0.8166 \Rightarrow B \approx 54.75°$$

$$A \approx 180° - 54.75° - 97.44° = 27.81°$$

22. Given: $C = 100°, a = 7, b = 4$

$c^2 = a^2 + b^2 - 2ab\cos C = 7^2 + 4^2 - 2(7)(4)\cos 100° \Rightarrow c \approx 8.64$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \approx \dfrac{16 + 8.64^2 - 49}{2(4)(8.64)} \approx 0.6036 \Rightarrow A \approx 52.95°$

$B \approx 180° - 52.95° - 100° = 27.05°$

23. Given: $a = 6, b = 9, c = 14$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{36 + 81 - 196}{2(6)(9)} \approx -0.7315 \Rightarrow C \approx 137.01°$

$\sin B = \dfrac{b\sin C}{c} \approx \dfrac{9\sin 137.01°}{14} \approx 0.4383 \Rightarrow B \approx 26.00°$

$A \approx 180° - 26.00° - 137.01° = 16.99°$

24. Given: $a = 75, b = 50, c = 110$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{75^2 + 50^2 - 110^2}{2(75)(50)} = -0.53 \Rightarrow C \approx 122.01°$

$\sin B = \dfrac{b\sin C}{c} \approx \dfrac{50\sin 122.01°}{110} \approx 0.3854 \Rightarrow B \approx 22.67°$

$A \approx 180° - 22.67° - 122.01° = 35.32°$

25. Given: $a = 2.5, b = 5.0, c = 4.5$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} = 0.0667 \Rightarrow B \approx 86.18°$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = 0.44 \Rightarrow C \approx 63.90°$

$A = 180° - B - C \approx 29.92°$

26. Given: $a = 16.4, b = 8.8, c = 12.2$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{8.8^2 + 12.2^2 - 16.4^2}{2(8.8)(12.2)} \approx -0.1988 \Rightarrow A \approx 101.47°$

$\sin B = \dfrac{b\sin A}{a} \approx \dfrac{8.8\sin 101.47°}{16.4} \approx 0.5259 \Rightarrow B \approx 31.73°$

$C \approx 180° - 101.47° - 31.73° = 46.80°$

27. Given: $B = 108°, a = 11, c = 11$

$b^2 = a^2 + c^2 - 2ac\cos B = 11^2 + 11^2 - 2(11)(11)\cos 108° \Rightarrow b \approx 17.80$

$A = C = \frac{1}{2}(180° - 108°) = 36°$

28. Given: $B = 150°, a = 10, c = 20$

$b^2 = 10^2 + 20^2 - 2(10)(20)\cos 150° \Rightarrow b \approx 29.09$

$\sin A = \dfrac{a\sin B}{b} \approx \dfrac{10\sin 150°}{29.09} \Rightarrow A \approx 9.90°$

$C \approx 180° - 150° - 9.90° = 20.10°$

29. Given: $C = 43°, a = 22.5, b = 31.4$

$c = \sqrt{a^2 + b^2 - 2ab\cos C} \approx 21.42$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} \approx -0.02169 \Rightarrow B \approx 91.24°$

$A = 180° - B - C \approx 45.76°$

30. Given: $A = 62°, b = 11.34, c = 19.52$

$a^2 = 11.34^2 + 19.52^2 - 2(11.34)(19.52)\cos 62° \Rightarrow a \approx 17.37$

$\sin B = \frac{b \sin A}{a} \approx \frac{11.34 \sin 62°}{17.37} \Rightarrow B \approx 35.20°$

$C \approx 180° - 62° - 35.20° = 82.80°$

31. Given: $C = 64°, b = 9, c = 13.$

Given two sides and an angle opposite one of them, the Law of Cosines cannot be used, so use the Law of Sines.

$\sin B = \frac{b \sin C}{c} = \frac{9 \sin 64°}{13} \approx 0.62224 \Rightarrow B \approx 38.48°$

$A \approx 180° - 38.48° - 64° = 77.52°$

$a = \frac{c \sin A}{\sin C} \approx \frac{13 \sin 77.52°}{\sin 64°} \approx 14.12$

32. Given: $a = 4, c = 5, B = 52°$

Given two sides and the included angle, the Law of Cosines can be used.

$b^2 = a^2 + c^2 - 2ac\cos B = 16 + 25 - 2(4)(5)\cos 52° \approx 16.38 \Rightarrow b \approx 4.05$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16 + 16.38 - 25}{2(4)(4.05)} \approx 0.22778 \Rightarrow C \approx 76.83°$

$A \approx 180° - 52° - 76.83° = 51.17°$

33. Given: $a = 13, b = 15, c = 24$

Given three sides, the Law of Cosines can be used.

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{169 + 225 - 576}{2(13)(15)} \approx -0.46667 \Rightarrow C \approx 117.82°$

$\sin A = \frac{a \sin C}{c} \approx \frac{13 \sin 117.82°}{24} \approx 0.47906 \Rightarrow A \approx 28.62°$

$B \approx 180° - 28.62° - 117.82° = 33.56°$

34. Given: $A = 44°, B = 31°, c = 2.8$

Given two angles and a side, the Law of Cosines cannot be used, so use the Law of Sines.

$C = 180° - 44° - 31° = 105°$

$a = \frac{c \sin A}{\sin C} = \frac{2.8 \sin 44°}{\sin 105°} \approx 2.01$

$b = \frac{c \sin B}{\sin C} = \frac{2.8 \sin 31°}{\sin 105°} \approx 1.49$

35. Given: $a = 160, B = 12°, C = 7°$

Given two angles and a side, use the Law of Sines.

$A = 180° - 12° - 7° = 161°$

$b = \frac{a \sin B}{\sin A} = \frac{160 \sin 12°}{\sin 161°} \approx 102.18$

$c = \frac{a \sin C}{\sin A} = \frac{160 \sin 7°}{\sin 161°} \approx 59.89$

36. Given: $A = 33°, b = 7, c = 10.$

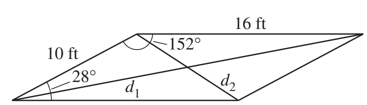
Given two sides and the included angle, use the Law of Cosines.

$a^2 = 7^2 + 10^2 - 2(7)(10)\cos 33° \Rightarrow a \approx 5.62$

$\sin B = \frac{b \sin A}{a} \approx \frac{7 \sin 33°}{5.62} \Rightarrow B \approx 42.72°$

$C \approx 180° - 33° - 42.72° = 104.28°$

37.



Let d_1 be the longer diagonal and d_2 be the shorter diagonal.

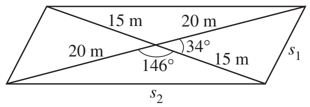
Using the Law of Cosines, you can find each of the diagonals.

$d_1^2 = 16^2 + 10^2 - 2(16)(10)\cos 152° \approx 638.543$

$d_1 \approx 25.27$ feet

$d_2^2 = 16^2 + 10^2 - 2(16)(10)\cos 28° \approx 73.457$

$d_2 \approx 8.57$ feet

38. 

$s_1^2 = 15^2 + 20^2 + 2 \cdot 15 \cdot 20 \cos 34° \approx 127.58$

$s_1 \approx 11.3$ meters

$s_2^2 = 15^2 + 20^2 + 2 \cdot 15 \cdot 20 \cos 146° \approx 1122.42$

$s_2 \approx 33.5$ meters

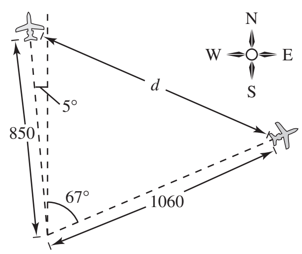
39. Length of $AC = \sqrt{300^2 + 425^2 - 2(300)(425)\cos 115°}$

≈ 615.1 meters

40. $d^2 = 850^2 + 1060^2 - 2(850)(1060)\cos 72°$

$\approx 1{,}289{,}251$

$d \approx 1135.5$ miles



41. $a = 3, b = 6, c = 8$

$$s = \frac{a+b+c}{2} = \frac{3+6+8}{2} = 8.5$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{8.5(5.5)(2.5)(0.5)} \approx 7.64$$

42. $a = 15, b = 8, c = 10$

$$s = \frac{a+b+c}{2} = \frac{15+8+10}{2} = 16.5$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{16.5(1.5)(8.5)(6.5)} \approx 36.98$$

43. $a = 12.3, b = 15.8, c = 3.7$

$$s = \frac{a+b+c}{2} = \frac{12.3+15.8+3.7}{2} = 15.9$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15.9(3.6)(0.1)(12.2)} = 8.36$$

44. $a = \frac{4}{5}, b = \frac{3}{4}, c = \frac{5}{8}$

$$s = \frac{a+b+c}{2} = \frac{\frac{4}{5}+\frac{3}{4}+\frac{5}{8}}{2} = \frac{87}{80}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{87}{80}\left(\frac{23}{80}\right)\left(\frac{27}{80}\right)\left(\frac{37}{80}\right)} \approx 0.22$$

45. $\|\mathbf{u}\| = \sqrt{(4-(-2))^2 + (6-1)^2} = \sqrt{61}$

$\|\mathbf{v}\| = \sqrt{(6-0)^2 + (3-(-2))^2} = \sqrt{61}$

u is directed along a line with a slope of $\frac{6-1}{4-(-2)} = \frac{5}{6}$.

v is directed along a line with a slope of $\frac{3-(-2)}{6-0} = \frac{5}{6}$.

Because **u** and **v** have identical magnitudes and directions, $\mathbf{u} = \mathbf{v}$.

46. $\|\mathbf{u}\| = \sqrt{(3-1)^2 + (-2-4)^2} = 2\sqrt{10}$

$\|\mathbf{v}\| = \sqrt{(-1-(-3))^2 + (-4-2)^2} = 2\sqrt{10}$

u is directed along a line with a slope of $\frac{-2-4}{3-1} = -3$.

v is directed along a line with a slope of $\frac{-4-2}{-1-(-3)} = -3$.

Because **u** and **v** have identical magnitudes and directions, $\mathbf{u} = \mathbf{v}$.

47. Initial point: $(-5, 4)$

Terminal point: $(2, -1)$

$\mathbf{v} = \langle 2-(-5), -1-4 \rangle = \langle 7, -5 \rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + (-5)^2} = \sqrt{74}$

48. Initial point: $(0, 1)$

Terminal point: $\left(6, \frac{7}{2}\right)$

$\mathbf{v} = \left\langle 6-0, \frac{7}{2}-1 \right\rangle = \left\langle 6, \frac{5}{2} \right\rangle$

$\|\mathbf{v}\| = \sqrt{6^2 + \left(\frac{5}{2}\right)^2} = \frac{13}{2}$

49. Initial point: $(0, 10)$

Terminal point: $(7, 3)$

$$\mathbf{v} = \langle 7 - 0, 3 - 10 \rangle = \langle 7, -7 \rangle$$

$$\|\mathbf{v}\| = \sqrt{7^2 + (-7)^2} = \sqrt{98} = 7\sqrt{2}$$

50. Initial point: $(1, 5)$

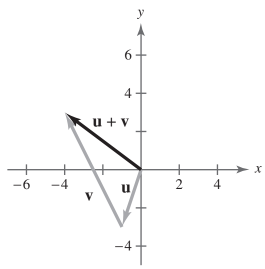
Terminal point: $(15, 9)$

$$\mathbf{v} = \langle 15 - 1, 9 - 5 \rangle = \langle 14, 4 \rangle$$

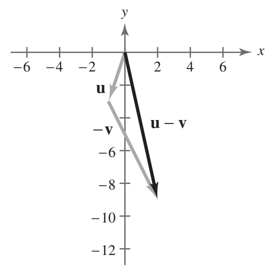
$$\|\mathbf{v}\| = \sqrt{14^2 + 4^2} = \sqrt{212} = 2\sqrt{53}$$

51. $\mathbf{u} = \langle -1, -3 \rangle, \mathbf{v} = \langle -3, 6 \rangle$

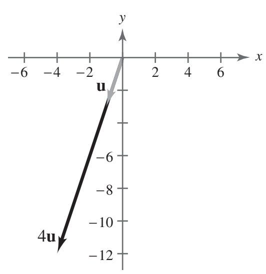
(a) $\mathbf{u} + \mathbf{v} = \langle -1, -3 \rangle + \langle -3, 6 \rangle = \langle -4, 3 \rangle$



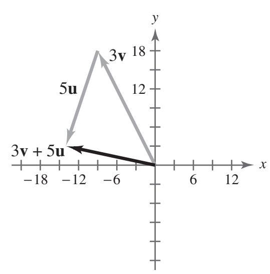
(b) $\mathbf{u} - \mathbf{v} = \langle -1, -3 \rangle - \langle -3, 6 \rangle = \langle 2, -9 \rangle$



(c) $4\mathbf{u} = 4\langle -1, -3 \rangle = \langle -4, -12 \rangle$

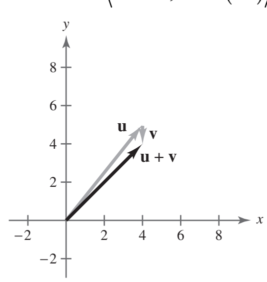


(d) $3\mathbf{v} + 5\mathbf{u} = 3\langle -3, 6 \rangle + 5\langle -1, -3 \rangle = \langle -9, 18 \rangle + \langle -5, -15 \rangle = \langle -14, 3 \rangle$

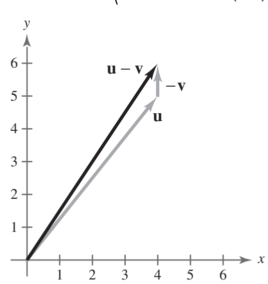


52. $\mathbf{u} = \langle 4, 5 \rangle, \mathbf{v} = \langle 0, -1 \rangle$

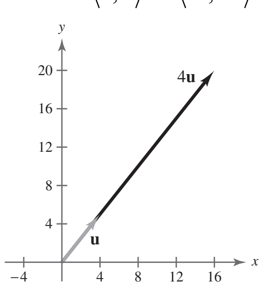
(a) $\mathbf{u} + \mathbf{v} = \langle 4 + 0, 5 + (-1) \rangle = \langle 4, 4 \rangle$



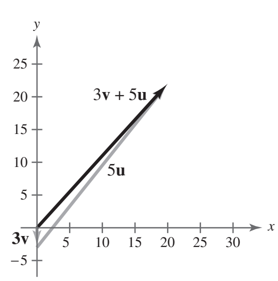
(b) $\mathbf{u} - \mathbf{v} = \langle 4 - 0, 5 - (-1) \rangle = \langle 4, 6 \rangle$



(c) $4\mathbf{u} = 4\langle 4, 5 \rangle = \langle 16, 20 \rangle$

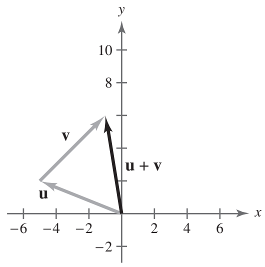


(d) $3\mathbf{v} + 5\mathbf{u} = 3\langle 0, -1 \rangle + 5\langle 4, 5 \rangle = \langle 0, -3 \rangle + \langle 20, 25 \rangle = \langle 20, 22 \rangle$

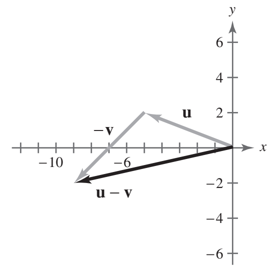


53. $\mathbf{u} = \langle -5, 2\rangle, \mathbf{v} = \langle 4, 4\rangle$

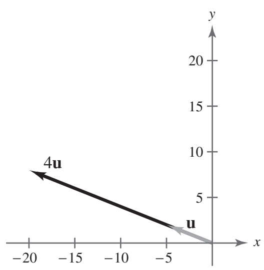
(a) $\mathbf{u} + \mathbf{v} = \langle -5, 2\rangle + \langle 4, 4\rangle = \langle -1, 6\rangle$



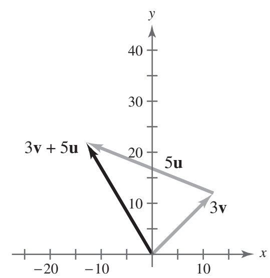
(b) $\mathbf{u} - \mathbf{v} = \langle -5, 2\rangle - \langle 4, 4\rangle = \langle -9, -2\rangle$



(c) $4\mathbf{u} = 4\langle -5, 2\rangle = \langle -20, 8\rangle$

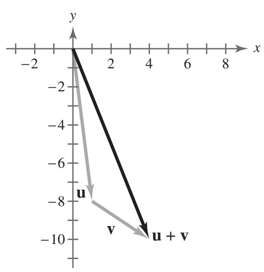


(d) $3\mathbf{v} + 5\mathbf{u} = 3\langle 4, 4\rangle + 5\langle -5, 2\rangle = \langle 12, 12\rangle + \langle -25, 10\rangle = \langle -13, 22\rangle$

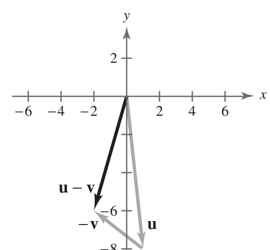


54. $\mathbf{u} = \langle 1, -8\rangle, \mathbf{v} = \langle 3, -2\rangle$

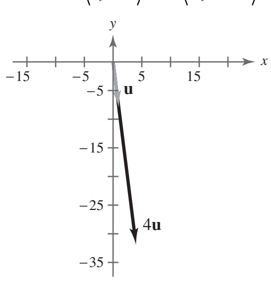
(a) $\mathbf{u} + \mathbf{v} = \langle 1 + 3, -8 + (-2)\rangle = \langle 4, -10\rangle$



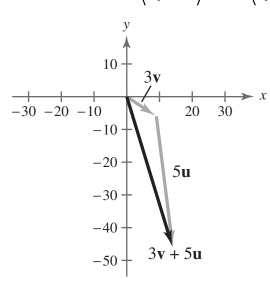
(b) $\mathbf{u} - \mathbf{v} = \langle 1 - 3, -8 - (-2)\rangle = \langle -2, -6\rangle$



(c) $4\mathbf{u} = 4\langle 1, -8\rangle = \langle 4, -32\rangle$

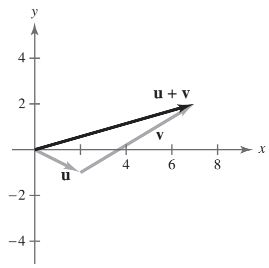


(d) $3\mathbf{v} + 5\mathbf{u} = 3\langle 3, -2\rangle + 5\langle 1, -8\rangle = \langle 9, -6\rangle + \langle 5, -40\rangle = \langle 14, -46\rangle$

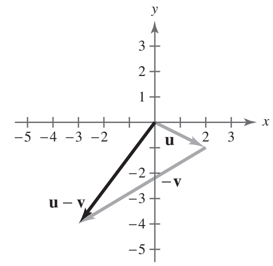


55. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$

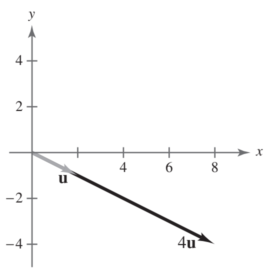
(a) $\mathbf{u} + \mathbf{v} = (2\mathbf{i} - \mathbf{j}) + (5\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} + 2\mathbf{j}$



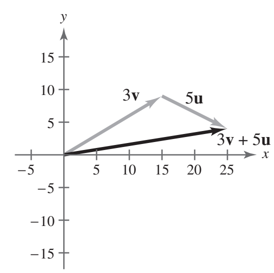
(b) $\mathbf{u} - \mathbf{v} = (2\mathbf{i} - \mathbf{j}) - (5\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} - 4\mathbf{j}$



(c) $4\mathbf{u} = 4(2\mathbf{i} - \mathbf{j}) = 8\mathbf{i} - 4\mathbf{j}$

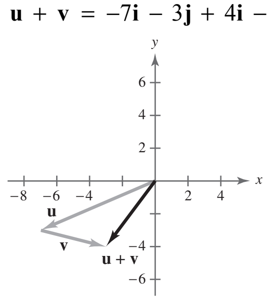


(d) $3\mathbf{v} + 5\mathbf{u} = 3(5\mathbf{i} + 3\mathbf{j}) + 5(2\mathbf{i} - \mathbf{j}) = 15\mathbf{i} + 9\mathbf{j} + 10\mathbf{i} - 5\mathbf{j} = 25\mathbf{i} + 4\mathbf{j}$

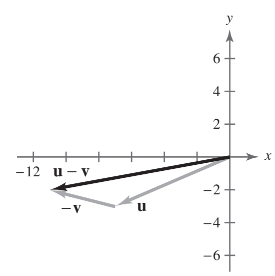


56. $\mathbf{u} = -7\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$

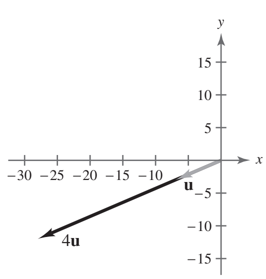
(a) $\mathbf{u} + \mathbf{v} = -7\mathbf{i} - 3\mathbf{j} + 4\mathbf{i} - \mathbf{j} = -3\mathbf{i} - 4\mathbf{j}$



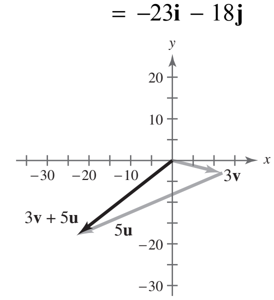
(b) $\mathbf{u} - \mathbf{v} = -7\mathbf{i} - 3\mathbf{j} - 4\mathbf{i} + \mathbf{j} = -11\mathbf{i} - 2\mathbf{j}$



(c) $4\mathbf{u} = 4(-7\mathbf{i} - 3\mathbf{j}) = -28\mathbf{i} - 12\mathbf{j}$

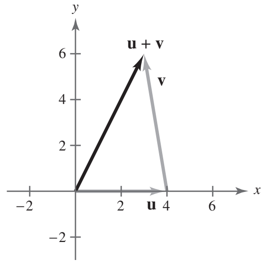


(d) $3\mathbf{v} + 5\mathbf{u} = 3(4\mathbf{i} - \mathbf{j}) + 5(-7\mathbf{i} - 3\mathbf{j})$
$= 12\mathbf{i} - 3\mathbf{j} - 35\mathbf{i} - 15\mathbf{j}$
$= -23\mathbf{i} - 18\mathbf{j}$

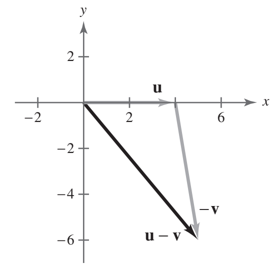


57. $\mathbf{u} = 4\mathbf{i}, \mathbf{v} = -\mathbf{i} + 6\mathbf{j}$

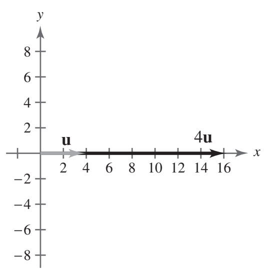
(a) $\mathbf{u} + \mathbf{v} = 4\mathbf{i} + (-\mathbf{i} + 6\mathbf{j}) = 3\mathbf{i} + 6\mathbf{j}$



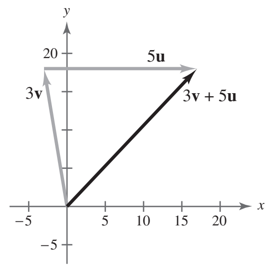
(b) $\mathbf{u} - \mathbf{v} = 4\mathbf{i} - (-\mathbf{i} + 6\mathbf{j}) = 5\mathbf{i} - 6\mathbf{j}$



(c) $4\mathbf{u} = 4(4\mathbf{i}) = 16\mathbf{i}$

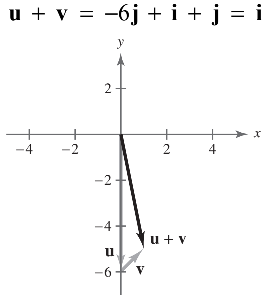


(d) $3\mathbf{v} + 5\mathbf{u} = 3(-\mathbf{i} + 6\mathbf{j}) + 5(4\mathbf{i}) = -3\mathbf{i} + 18\mathbf{j} + 20\mathbf{i} = 17\mathbf{i} + 18\mathbf{j}$

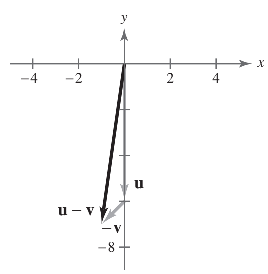


58. $\mathbf{u} = -6\mathbf{j}, \mathbf{v} = \mathbf{i} + \mathbf{j}$

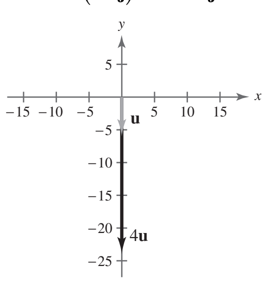
(a) $\mathbf{u} + \mathbf{v} = -6\mathbf{j} + \mathbf{i} + \mathbf{j} = \mathbf{i} - 5\mathbf{j}$



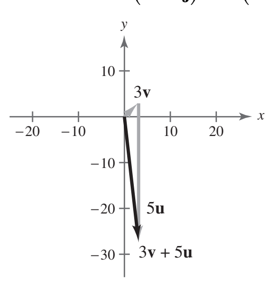
(b) $\mathbf{u} - \mathbf{v} = -6\mathbf{j} - \mathbf{i} - \mathbf{j} = -\mathbf{i} - 7\mathbf{j}$



(c) $4\mathbf{u} = 4(-6\mathbf{j}) = -24\mathbf{j}$



(d) $3\mathbf{v} + 5\mathbf{u} = 3(\mathbf{i} + \mathbf{j}) + 5(-6\mathbf{j}) = 3\mathbf{i} + 3\mathbf{j} - 30\mathbf{j} = 3\mathbf{i} - 27\mathbf{j}$



59. $P = (2, 3), Q = (1, 8)$

$\overrightarrow{PQ} = \mathbf{v} = \langle 1 - 2, 8 - 3\rangle$

$\mathbf{v} = \langle -1, 5\rangle$

$\mathbf{v} = -\mathbf{i} + 5\mathbf{j}$

60. $P = (4, -2), Q = (-2, -10)$

$\overrightarrow{PQ} = \mathbf{v} = \langle -2 - 4, -10 - (-2)\rangle$

$\mathbf{v} = \langle -6, -8\rangle$

$\mathbf{v} = -6\mathbf{i} - 8\mathbf{j}$

61. $P = (3, 4), Q = (9, 8)$

$\overrightarrow{PQ} = \mathbf{v} = \langle 9 - 3, 8 - 4\rangle$

$\mathbf{v} = \langle 6, 4\rangle$

$\mathbf{v} = 6\mathbf{i} + 4\mathbf{j}$

62. $P = (-2, 7), Q = (5, -9)$

$\overrightarrow{PQ} = \mathbf{v} = \langle 5 - (-2), -9 - 7\rangle$

$\mathbf{v} = \langle 7, -16\rangle$

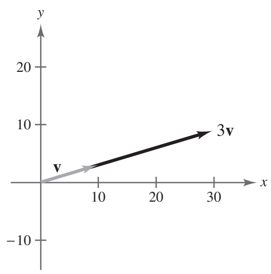
$\mathbf{v} = 7\mathbf{i} - 16\mathbf{j}$

63. $\mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$3\mathbf{v} = 3(10\mathbf{i} + 3\mathbf{j})$

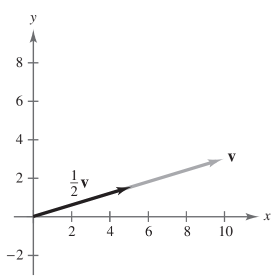
$= 30\mathbf{i} + 9\mathbf{j}$

$= \langle 30, 9\rangle$



64. $\mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$\frac{1}{2}\mathbf{v} = 5\mathbf{i} + \frac{3}{2}\mathbf{j} = \left\langle 5, \frac{3}{2}\right\rangle$

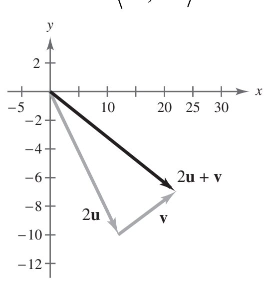


65. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$2\mathbf{u} + \mathbf{v} = 2(6\mathbf{i} - 5\mathbf{j}) + (10\mathbf{i} + 3\mathbf{j})$

$= 22\mathbf{i} - 7\mathbf{j}$

$= \langle 22, -7\rangle$

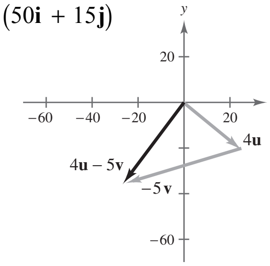


66. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$4\mathbf{u} - 5\mathbf{v} = (24\mathbf{i} - 20\mathbf{j}) - (50\mathbf{i} + 15\mathbf{j})$

$= -26\mathbf{i} - 35\mathbf{j}$

$= \langle -26, -35\rangle$



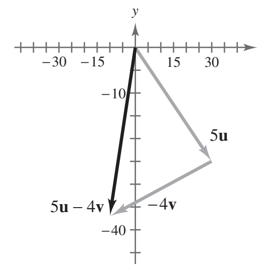
67. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$5\mathbf{u} - 4\mathbf{v} = 5(6\mathbf{i} - 5\mathbf{j}) - 4(10\mathbf{i} + 3\mathbf{j})$

$= 30\mathbf{i} - 25\mathbf{j} - 40\mathbf{i} - 12\mathbf{j}$

$= -10\mathbf{i} - 37\mathbf{j}$

$= \langle -10, -37\rangle$



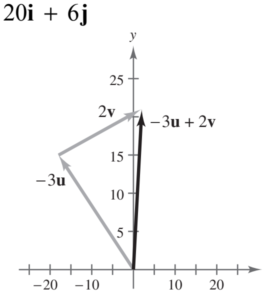
68. $\mathbf{u} = 6\mathbf{i} - 5\mathbf{j}, \mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$-3\mathbf{u} + 2\mathbf{v} = -3(6\mathbf{i} - 5\mathbf{j}) + 2(10\mathbf{i} + 3\mathbf{j})$

$= -18\mathbf{i} + 15\mathbf{j} + 20\mathbf{i} + 6\mathbf{j}$

$= 2\mathbf{i} + 21\mathbf{j}$

$= \langle 2, 21\rangle$



69. $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{5^2 + 4^2} = \sqrt{41}$

$\tan\theta = \frac{4}{5} \Rightarrow \theta \approx 38.7°$

70. $\mathbf{v} = -4\mathbf{i} + 7\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-4)^2 + 7^2} = \sqrt{65}$

$\tan\theta = \dfrac{7}{-4}, \theta$ in Quadrant II $\Rightarrow \theta \approx 119.7°$

71. $\mathbf{v} = -3\mathbf{i} - 3\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$

$\tan\theta = \frac{-3}{-3} = 1 \Rightarrow \theta = 225°$

72. $\mathbf{v} = 8\mathbf{i} - \mathbf{j}$

$\|\mathbf{v}\| = \sqrt{8^2 + (-1)^2} = \sqrt{65}$

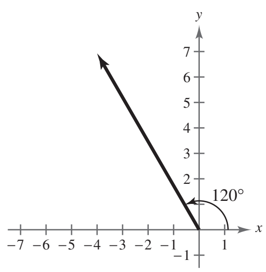
$\tan\theta = \frac{-1}{8}$, θ in Quadrant IV $\Rightarrow \theta \approx 352.9°$

73. $\mathbf{v} = 8(\cos 120° + i \sin 120°)$

$= 8\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= -4 + 4\sqrt{3}i$

$= \langle -4, 4\sqrt{3} \rangle$

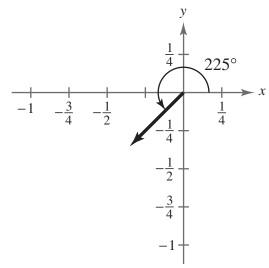


74. $\mathbf{v} = \frac{1}{2}(\cos 225° + i \sin 225°)$

$= \frac{1}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{4}i$

$= \left\langle -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4} \right\rangle$



75. Rope One:

$\mathbf{u} = \|\mathbf{u}\|(\cos 30°\mathbf{i} - \sin 30°\mathbf{j}) = \|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$

Rope Two:

$\mathbf{v} = \|\mathbf{u}\|(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j}) = \|\mathbf{u}\|\left(-\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$

Resultant: $\mathbf{u} + \mathbf{v} = -\|\mathbf{u}\|\mathbf{j} = -180\mathbf{j}$

$\|\mathbf{u}\| = 180$

So, the tension on each rope is $\|\mathbf{u}\| = 180$ lb.

76. Force One:

$$\mathbf{u} = 85(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$$
$$= 85\left(\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}\right)$$
$$= \frac{85\sqrt{2}}{2}\mathbf{i} + \frac{85\sqrt{2}}{2}\mathbf{j}$$

Force Two:

$$\mathbf{v} = 50(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$$
$$= 50\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right)$$
$$= 25\mathbf{i} + 25\sqrt{3}\mathbf{j}$$

Resultant Force: $\mathbf{u} + \mathbf{v} = \left(\frac{85\sqrt{2}}{2} + 25\right)\mathbf{i} + \left(\frac{85\sqrt{2}}{2} + 25\sqrt{3}\right)\mathbf{j}$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{\left(\frac{85\sqrt{2}}{2} + 25\right)^2 + \left(\frac{85\sqrt{2}}{2} + 25\sqrt{3}\right)^2}$$
$$\approx 133.92 \text{ pounds}$$

$$\tan\theta = \frac{\frac{85\sqrt{2}}{2} + 25\sqrt{3}}{\frac{85\sqrt{2}}{2} + 25}$$
$$\theta = 50.5°$$

77. $\mathbf{u} = \langle 6, 7\rangle, \mathbf{v} = \langle -3, 9\rangle$

$\mathbf{u} \cdot \mathbf{v} = 6(-3) + 7(9) = 45$

78. $\mathbf{u} = \langle -7, 12\rangle, \mathbf{v} = \langle -4, -14\rangle$

$\mathbf{u} \cdot \mathbf{v} = -7(-4) + 12(-14) = -140$

79. $\mathbf{u} = 3\mathbf{i} + 7\mathbf{j}, \mathbf{v} = 11\mathbf{i} - 5\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = 3(11) + 7(-5) = -2$

80. $\mathbf{u} = -7\mathbf{i} + 2\mathbf{j}, \mathbf{v} = 16\mathbf{i} - 12\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = -7(16) + 2(-12) = -136$

81. $\mathbf{u} = \langle -4, 2\rangle$

$2\mathbf{u} = \langle -8, 4\rangle$

$2\mathbf{u} \cdot \mathbf{u} = -8(-4) + 4(2) = 40$

The result is a scalar.

82. $\mathbf{u} = \langle -4, 2\rangle, \mathbf{v} = \langle 5, 1\rangle$

$3\mathbf{u} = \langle -12, 6\rangle$

$3\mathbf{u} \cdot \mathbf{v} = -12(5) + 6(1) = -54$

The result is a scalar.

83. $\mathbf{u} = \langle -4, 2\rangle$

$4 - \|\mathbf{u}\| = 4 - \sqrt{(-4)^2 + 2^2} = 4 - \sqrt{20} = 4 - 2\sqrt{5}$

The result is a scalar.

84. $\mathbf{v} = \langle 5, 1\rangle$

$\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v} = 5(5) + 1(1) = 26$

The result is a scalar.

85. $\mathbf{u} = \langle -4, 2\rangle, \mathbf{v} = \langle 5, 1\rangle$

$$\mathbf{u}(\mathbf{u} \cdot \mathbf{v}) = \langle -4, 2\rangle[-4(5) + 2(1)]$$
$$= -18\langle -4, 2\rangle$$
$$= \langle 72, -36\rangle$$

The result is a vector.

86. $\mathbf{u} = \langle -4, 2\rangle, \mathbf{v} = \langle 5, 1\rangle$

$$(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = [-4(5) + 2(1)]\langle 5, 1\rangle$$
$$= -18\langle 5, 1\rangle$$
$$= \langle -90, -18\rangle$$

The result is a vector.

87. $\mathbf{u} = \langle -4, 2\rangle, \mathbf{v} = \langle 5, 1\rangle$

$$\begin{aligned}(\mathbf{u}\cdot\mathbf{u}) - (\mathbf{u}\cdot\mathbf{v}) &= \left[-4(-4) + 2(2)\right] - \left[-4(5) + 2(1)\right]\\ &= 20 - (-18)\\ &= 38\end{aligned}$$

The result is a scalar.

88. $\mathbf{u} = \langle -4, 2\rangle, \mathbf{v} = \langle 5, 1\rangle$

$$\begin{aligned}(\mathbf{v}\cdot\mathbf{v}) - (\mathbf{v}\cdot\mathbf{u}) &= \left[5(5) + 1(1)\right] - \left[5(-4) + 1(2)\right]\\ &= 26 - (-18)\\ &= 44\end{aligned}$$

The result is a scalar.

89. $\mathbf{u} = \cos\frac{7\pi}{4}\mathbf{i} + \sin\frac{7\pi}{4}\mathbf{j} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\rangle$

$\mathbf{v} = \cos\frac{5\pi}{6}\mathbf{i} + \sin\frac{5\pi}{6}\mathbf{j} = \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle$

$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-\sqrt{3} - 1}{2\sqrt{2}} \Rightarrow \theta = \frac{11\pi}{12}$

90. $\mathbf{u} = \cos 45^\circ\mathbf{i} + \sin 45^\circ\mathbf{j}$

$\mathbf{v} = \cos 300^\circ\mathbf{i} + \sin 300^\circ\mathbf{j}$

Angle between **u** and **v**: $60^\circ + 45^\circ = 105^\circ$

91. $\mathbf{u} = \left\langle 2\sqrt{2}, -4\right\rangle, \mathbf{v} = \left\langle -\sqrt{2}, 1\right\rangle$

$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-8}{\left(\sqrt{24}\right)\left(\sqrt{3}\right)} \Rightarrow \theta \approx 160.5^\circ$

92. $\mathbf{u} = \left\langle 3, \sqrt{3}\right\rangle, \mathbf{v} = \left\langle 4, 3\sqrt{3}\right\rangle$

$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{21}{\sqrt{12}\sqrt{43}} \Rightarrow \theta \approx 22.4^\circ$

93. $\mathbf{u} = \langle -3, 8\rangle$

$\mathbf{v} = \langle 8, 3\rangle$

$\mathbf{u}\cdot\mathbf{v} = -3(8) + 8(3) = 0$

u and **v** are orthogonal.

94. $\mathbf{u} = \left\langle \frac{1}{4}, -\frac{1}{2}\right\rangle, \mathbf{v} = \langle -2, 4\rangle$

$\mathbf{v} = -8\mathbf{u} \Rightarrow$ Parallel

u and **v** are *not* orthogonal.

95. $\mathbf{u} = -\mathbf{i}$

$\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} \neq 0$

u and **v** are *not* orthogonal.

96. $\mathbf{u} = -2\mathbf{i} + \mathbf{j}, \mathbf{v} = 3\mathbf{i} + 6\mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = 0$

u and **v** are orthogonal.

97. $\mathbf{u} = \langle -4, 3\rangle, \mathbf{v} = \langle -8, -2\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{26}{68}\right)\langle -8, -2\rangle = -\frac{13}{17}\langle 4, 1\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -4, 3\rangle - \left(-\frac{13}{17}\right)\langle 4, 1\rangle = \frac{16}{17}\langle -1, 4\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = -\frac{13}{17}\langle 4, 1\rangle + \frac{16}{17}\langle -1, 4\rangle$

98. $\mathbf{u} = \langle 5, 6\rangle, \mathbf{v} = \langle 10, 0\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{50}{100}\langle 10, 0\rangle = \langle 5, 0\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 5, 6\rangle - \langle 5, 0\rangle = \langle 0, 6\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \langle 5, 0\rangle + \langle 0, 6\rangle$

99. $\mathbf{u} = \langle 2, 7\rangle, \mathbf{v} = \langle 1, -1\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = -\frac{5}{2}\langle 1, -1\rangle = \frac{5}{2}\langle -1, 1\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 7\rangle - \left(\frac{5}{2}\right)\langle -1, 1\rangle = \frac{9}{2}\langle 1, 1\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{5}{2}\langle -1, 1\rangle + \frac{9}{2}\langle 1, 1\rangle$

100. $\mathbf{u} = \langle -3, 5\rangle, \mathbf{v} = \langle -5, 2\rangle$

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{25}{29}\langle -5, 2\rangle$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -3, 5\rangle - \frac{25}{29}\langle -5, 2\rangle = \frac{19}{29}\langle 2, 5\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{25}{29}\langle -5, 2\rangle + \frac{19}{25}\langle 2, 5\rangle$

101. $P = (5, 3), Q = (8, 9) \Rightarrow \overrightarrow{PQ} = \langle 3, 6\rangle$

$\text{Work} = \mathbf{v}\cdot\overrightarrow{PQ} = \langle 2, 7\rangle\cdot\langle 3, 6\rangle = 48$

102.
$$\begin{aligned}\text{Work} &= \mathbf{v}\cdot\overrightarrow{PQ}\\ &= (3\mathbf{i} - 6\mathbf{j})\cdot(-10\mathbf{i} + 17\mathbf{j})\\ &= -30 - 102\\ &= -132\end{aligned}$$

103. $\text{Work} = (18{,}000)\left(\frac{48}{12}\right) = 72{,}000$ foot-pounds

104. $\text{Work} = \cos\theta\|\mathbf{F}\|\|\overrightarrow{PQ}\|$
$= (\cos 20°)(25 \text{ pounds})(12 \text{ feet})$
$= 281.9 \text{ foot-pounds}$

105. True. Using the Law of Cosines, there is exactly one solution.

106. False. The triangle cannot be solved if only three angles are known.

107. True. $\sin 90°$ is defined in the Law of Sines.

108. False. There may be no solution, one solution, or two solutions.

109. **A** and **C** appear to have the same magnitude and direction.

110. $\|\mathbf{u} + \mathbf{v}\|$ is larger in figure (a) because the angle between **u** and **v** is acute rather than obtuse.

111. If $k > 0$, the direction of $k\mathbf{u}$ is the same, and the magnitude is $k\|\mathbf{u}\|$.

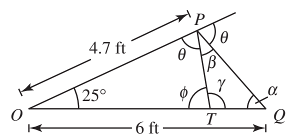
If $k < 0$, the direction of $k\mathbf{u}$ is the opposite direction of **u**, and the magnitude is $|k|\ \|\mathbf{u}\|$.

112. When u and v have the same initial point, $\mathbf{u} - \mathbf{v}$ is the vector from the terminal point of v to the terminal point of u.

113. A vector in the plane has both a magnitude and a direction.

Problem Solving for Chapter 3

1. $\left(\overrightarrow{PQ}\right)^2 = 4.7^2 + 6^2 - 2(4.7)(6)\cos 25°A$



$\overrightarrow{PQ} \approx 2.6409 \text{ feet}$

$\dfrac{\sin\alpha}{4.7} = \dfrac{\sin 25°}{2.6409} \Rightarrow \alpha \approx 48.78°$

$\theta + \beta = 180° - 25° - 48.78° = 106.22°$

$(\theta + \beta) + \theta = 180° \Rightarrow \theta = 180° - 106.22° = 73.78°$

$\beta = 106.22° - 73.78° = 32.44°$

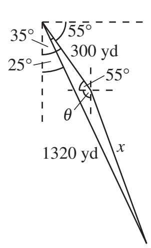
$\gamma = 180° - \alpha - \beta = 180° - 48.78° - 32.44° = 98.78°$

$\phi = 180° - \gamma = 180° - 98.78° = 81.22°$

$\dfrac{\overrightarrow{PT}}{\sin 25°} = \dfrac{4.7}{\sin 81.22°}$

$\overrightarrow{PT} \approx 2.01 \text{ feet}$

2.



$\dfrac{3}{4} \text{ mile} = 1320 \text{ yards}$

$x^2 = 1320^2 + 300^2 - 2(1320)(300)\cos 10°$

$x \approx 1025.88 \text{ yards} \approx 0.58 \text{ mile}$

$\dfrac{\sin\theta}{1320} = \dfrac{\sin 10°}{1025.881}$

$\sin\theta \approx 0.2234$

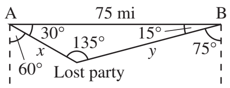
$\theta = 180° - \sin^{-1}(0.2234)$

$\theta \approx 167.09°$

Bearing: $\theta - 55° - 90° \approx 22.09°$

S 22.09° E

3. (a)



(b) $\dfrac{x}{\sin 15°} = \dfrac{75}{\sin 135°}$

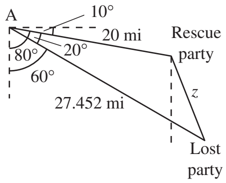
$x \approx 27.45$ miles

and

$\dfrac{y}{\sin 30°} = \dfrac{75}{\sin 135°}$

$y \approx 53.03$ miles

(c)



$z^2 = (27.45)^2 + (20)^2 - 2(27.45)(20)\cos 20°$

$z \approx 11.03$ miles

$\dfrac{\sin\theta}{27.45} = \dfrac{\sin 20°}{11.03}$

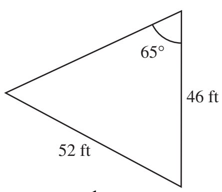
$\sin\theta \approx 0.8511$

$\theta = 180° - \sin^{-1}(0.8511)$

$\theta \approx 121.7°$

To find the bearing, we have $\theta - 10° - 90° \approx 21.7°$. Bearing: S 21.7° E

4. (a)



(b) $\dfrac{\sin C}{46} = \dfrac{\sin 65°}{52}$

$\sin C = \dfrac{46\sin 65°}{52} \approx 0.801734$

$C \approx 53.296°$

$A = 180° - B - C = 61.704°$

$\dfrac{a}{\sin 61.704°} = \dfrac{52}{\sin 65°}$

$a = \dfrac{52\sin 61.704°}{\sin 65°}$

$a \approx 50.5$ feet

(c) Area $= \dfrac{1}{2}(46)(52)\sin 61.704°$

≈ 1053.09 square feet

Number of bags: $\dfrac{1053.09}{50} \approx 21.06$

To entirely cover the courtyard, you would need to buy 22 bags.

5. If $\mathbf{u} \neq 0$, $\mathbf{v} \neq 0$, and $\mathbf{u} + \mathbf{v} \neq 0$, then $\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = \left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = \left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$ because all of these are magnitudes of unit vectors.

(a) $\mathbf{u} = \langle 1, -1\rangle$, $\mathbf{v} = \langle -1, 2\rangle$, $\mathbf{u} + \mathbf{v} = \langle 0, 1\rangle$

(i) $\|\mathbf{u}\| = \sqrt{2}$ (ii) $\|\mathbf{v}\| = \sqrt{5}$ (iii) $\|\mathbf{u}+\mathbf{v}\| = 1$ (iv) $\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$ (v) $\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$ (vi) $\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

(b) $\mathbf{u} = \langle 0, 1\rangle$, $\mathbf{v} = \langle 3, -3\rangle$, $\mathbf{u} + \mathbf{v} = \langle 3, -2\rangle$

(i) $\|\mathbf{u}\| = 1$ (ii) $\|\mathbf{v}\| = \sqrt{18} = 3\sqrt{2}$ (iii) $\|\mathbf{u}+\mathbf{v}\| = \sqrt{13}$ (iv) $\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$ (v) $\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$ (vi) $\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

(c) $\mathbf{u} = \left\langle 1, \dfrac{1}{2}\right\rangle$, $\mathbf{v} = \langle 2, 3\rangle$, $\mathbf{u} + \mathbf{v} = \left\langle 3, \dfrac{7}{2}\right\rangle$

(i) $\|\mathbf{u}\| = \dfrac{\sqrt{5}}{2}$ (ii) $\|\mathbf{v}\| = \sqrt{13}$ (iii) $\|\mathbf{u}+\mathbf{v}\| = \sqrt{9 + \dfrac{49}{4}} = \dfrac{\sqrt{85}}{2}$ (iv) $\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(v) $\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$ (vi) $\left\|\dfrac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\| = 1$

(d) $\mathbf{u} = \langle 2, -4 \rangle, \quad \mathbf{v} = \langle 5, 5 \rangle, \quad \mathbf{u} + \mathbf{v} = \langle 7, 1 \rangle$

(i) $\|\mathbf{u}\| = \sqrt{20} = 2\sqrt{5}$ (ii) $\|\mathbf{v}\| = \sqrt{50} = 5\sqrt{2}$ (iii) $\|\mathbf{u} + \mathbf{v}\| = \sqrt{50} = 5\sqrt{2}$ (iv) $\left\|\dfrac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(v) $\left\|\dfrac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$ (vi) $\left\|\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|}\right\| = 1$

6. Initial point: $(0, 0)$

Terminal point: $\left(\dfrac{\mathbf{u}_1 + \mathbf{v}_1}{2}, \dfrac{\mathbf{u}_2 + \mathbf{v}_2}{2}\right)$

$$\mathbf{w} = \left\langle \frac{\mathbf{u}_1 + \mathbf{v}_1}{2}, \frac{\mathbf{u}_2 + \mathbf{v}_2}{2} \right\rangle = \frac{1}{2}(\mathbf{u} + \mathbf{v})$$

Initial point: $(\mathbf{u}_1, \mathbf{u}_2)$

Terminal point: $\dfrac{1}{2}(\mathbf{u}_1 + \mathbf{v}_1, \mathbf{u}_2 + \mathbf{v}_2)$

$$\mathbf{w} = \left\langle \frac{\mathbf{u}_1 + \mathbf{v}_1}{2} - \mathbf{u}_1, \frac{\mathbf{u}_2 + \mathbf{v}_2}{2} - \mathbf{u}_2 \right\rangle$$

$$= \left\langle \frac{\mathbf{v}_1 - \mathbf{u}_1}{2}, \frac{\mathbf{v}_2 - \mathbf{u}_2}{2} \right\rangle = \frac{1}{2}(\mathbf{v} - \mathbf{u})$$

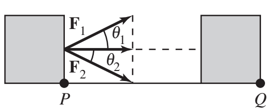
7. (a) The angle between them is $0°$.

(b) The angle between them is $180°$.

(c) No. At most it can be equal to the sum when the angle between them is $0°$.

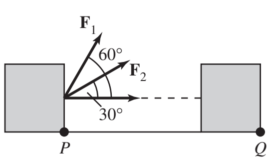
8. $W = (\cos\theta)\|\mathbf{F}\|\left\|\overrightarrow{PQ}\right\|$ and $\|\mathbf{F}_1\| = \|\mathbf{F}_2\|$

(a)



If $\theta_1 = -\theta_2$ then the work is the same because $\cos(-\theta) = \cos\theta$.

(b)



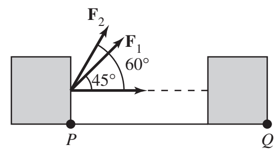
If $\theta_1 = 60°$ then $W_1 = \dfrac{1}{2}\|\mathbf{F}_1\|\left\|\overrightarrow{PQ}\right\|$.

If $\theta_2 = 30°$ then $W_2 = \dfrac{\sqrt{3}}{2}\|\mathbf{F}_2\|\left\|\overrightarrow{PQ}\right\|$.

$W_2 = \sqrt{3}W_1$

The amount of work done by $\mathbf{F}_2$ is $\sqrt{3}$ times as great as the amount of work done by $\mathbf{F}_1$.

(c)



If $\theta_1 = 45°$, then $W_1 = \dfrac{\sqrt{2}}{2}\|\mathrm{F}_1\|\left\|\overrightarrow{PQ}\right\|$.

If $\theta_2 = 60°$, then $W_2 = \dfrac{1}{2}\|\mathrm{F}_2\|\left\|\overrightarrow{PQ}\right\|$.

$W_2 = \sqrt{2}\,W_1$

The amount of work done by F_2 is $\sqrt{2}$ times as great as the amount of work done by F_1.

9. (a) Since $0 < C < 180°$, $\cos\left(\dfrac{C}{2}\right) = \sqrt{\dfrac{1 + \cos C}{2}}$.

Hence,

$$\cos\left(\frac{C}{2}\right) = \sqrt{\frac{1 + \left(a^2 + b^2 - c^2\right)/2ab}{2}} = \sqrt{\frac{2ab + a^2 + b^2 - c^2}{4ab}}.$$

On the other hand,

$$s(s-c) = \frac{1}{2}(a+b+c)\left(\frac{1}{2}(a+b+c) - c\right)$$

$$= \frac{1}{2}(a+b+c)\frac{1}{2}(a+b-c)$$

$$= \frac{1}{4}((a+b)^2 - c^2)$$

$$= \frac{1}{4}(a^2 + b^2 + 2ab - c^2).$$

Thus, $\sqrt{\dfrac{s(s-c)}{ab}} = \sqrt{\dfrac{a^2 + b^2 + 2ab - c^2}{4ab}}$ and we have verified that $\cos\left(\dfrac{C}{2}\right) = \sqrt{\dfrac{s(s-c)}{ab}}$.

(b) Since $0 < C < 180°$, $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{1-\cos C}{2}}$.

Hence,

$$\sin\left(\frac{C}{2}\right) = \sqrt{\frac{1-(a^2+b^2-c^2)/(2ab)}{2}} = \sqrt{\frac{2ab-a^2-b^2+c^2}{4ab}}.$$

On the other hand,

$$\begin{aligned}(s-a)(s-b) &= \left[\frac{1}{2}(a+b+c)-a\right]\left[\frac{1}{2}(a+b+c)-b\right]\\ &= \frac{1}{2}(b+c-a)\frac{1}{2}(a+c-b)\\ &= \frac{1}{4}[c-(a-b)][c+(a-b)]\\ &= \frac{1}{4}[c^2-(a-b)^2]\\ &= \frac{1}{4}(c^2-a^2-b^2+2ab).\end{aligned}$$

Thus, $\sqrt{\frac{(s-a)(s-b)}{ab}} = \sqrt{\frac{c^2-a^2-b^2+2ab}{4ab}} = \sin\left(\frac{C}{2}\right)$.

10. Let $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \cdot \mathbf{w} = 0$.

Then,
$$\begin{aligned}\mathbf{u} \cdot (c\mathbf{v} + d\mathbf{w}) &= \mathbf{u} \cdot c\mathbf{v} + \mathbf{u} \cdot d\mathbf{w}\\ &= c(\mathbf{u} \cdot \mathbf{v}) + d(\mathbf{u} \cdot \mathbf{w})\\ &= c(0) + d(0)\\ &= 0.\end{aligned}$$

So for all scalars c and d, $\mathbf{u}$ is orthogonal to $c\mathbf{v} + d\mathbf{w}$.

11. Answers will vary.

12. (a)
$$\begin{aligned}\cos\theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\\ \cos\theta &= \frac{\langle 8, -4\rangle \cdot \langle 2, 5\rangle}{\sqrt{80} \cdot \sqrt{29}}\\ \cos\theta &= \frac{-4}{\sqrt{2320}}\\ \theta &\approx 94.76°\end{aligned}$$

$$\begin{aligned}\text{proj}_{\mathbf{v}}\,\mathbf{u} &= \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\langle 2, 5\rangle\\ &= \left(\frac{-4}{\left(\sqrt{29}\right)^2}\right)\langle 2, 5\rangle\\ &= \left\langle -\frac{8}{29}, -\frac{20}{29}\right\rangle\end{aligned}$$

(b)
$$\begin{aligned}\cos\theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\\ \cos\theta &= \frac{\langle 2, -6\rangle \cdot \langle 4, 1\rangle}{\sqrt{40} \cdot \sqrt{17}}\\ \cos\theta &= \frac{2}{\sqrt{680}}\\ \theta &\approx 85.60°\end{aligned}$$

$$\begin{aligned}\text{proj}_{\mathbf{v}}\,\mathbf{u} &= \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}\\ &= \left(\frac{2}{\left(\sqrt{17}\right)^2}\right)\langle 4, 1\rangle\\ &= \left\langle \frac{8}{17}, \frac{2}{17}\right\rangle\end{aligned}$$

(c)
$$\begin{aligned}\cos\theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\\ \cos\theta &= \frac{\langle 5, 6\rangle \cdot \langle -1, 3\rangle}{\sqrt{61} \cdot \sqrt{10}}\\ \cos\theta &= \frac{13}{\sqrt{610}}\\ \theta &\approx 58.24°\end{aligned}$$

$$\begin{aligned}\text{proj}_{\mathbf{v}}\,\mathbf{u} &= \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}\\ &= \left(\frac{13}{\left(\sqrt{10}\right)^2}\right)\langle -1, 3\rangle\\ &= \left\langle -\frac{13}{10}, \frac{39}{10}\right\rangle\end{aligned}$$

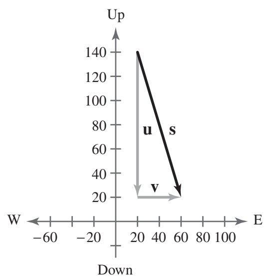
(d)
$$\begin{aligned}\cos\theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\\ \cos\theta &= \frac{\langle 3, -2\rangle \cdot \langle -2, 1\rangle}{\sqrt{13} \cdot \sqrt{5}}\\ \cos\theta &= \frac{-8}{\sqrt{65}}\\ \theta &\approx 172.87°\end{aligned}$$

$$\begin{aligned}\text{proj}_{\mathbf{v}}\,\mathbf{u} &= \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}\\ &= \left(\frac{-8}{\left(\sqrt{5}\right)^2}\right)\langle -2, 1\rangle\\ &= \left\langle \frac{16}{5}, -\frac{8}{5}\right\rangle\end{aligned}$$

13. (a) $\mathbf{u} = -120\mathbf{j}$

$\mathbf{v} = 40\mathbf{i}$

(b) $\mathbf{s} = \mathbf{u} + \mathbf{v} = 40\mathbf{i} - 120\mathbf{j}$



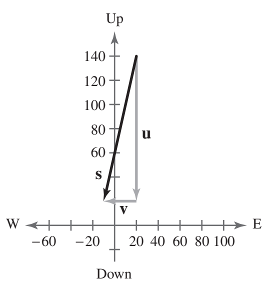
(c) $\|\mathbf{s}\| = \sqrt{40^2 + (-120)^2} = \sqrt{16{,}000} = 40\sqrt{10}$

≈ 126.5 miles per hour

This represents the actual rate of the skydiver's fall.

(d) $\tan\theta = \frac{120}{40} \Rightarrow \theta = \tan^{-1} 3 \Rightarrow \theta \approx 71.565°$

(e)



$\mathbf{s} = 30\mathbf{i} - 120\mathbf{j}$

$\|\mathbf{s}\| = \sqrt{30^2 + (-120)^2}$

$= \sqrt{15{,}300}$

≈ 123.7 miles per hour

14. (a)

θ	$100 \sin\theta$	$100 \cos\theta$
0.5°	0.873	99.996
1.0°	1.745	99.985
1.5°	2.618	99.966
2.0°	3.490	99.939
2.5°	4.362	99.905
3.0°	5.234	99.863

(b) No, the airplane's speed does *not* equal the sum of the vertical and horizontal components of its velocity.

To find speed:

$$\text{speed} = \sqrt{(\|\mathbf{v}\| \sin\theta)^2 + (\|\mathbf{v}\| \cos\theta)^2}$$

(c) (i) speed $= \sqrt{5.235^2 + 149.909^2}$

≈ 150 miles per hour

(ii) speed $= \sqrt{10.463^2 + 149.634^2}$

≈ 150 miles per hour

Practice Test for Chapter 3

For Exercises 1 and 2, use the Law of Sines to find the remaining sides and angles of the triangle.

1. $A = 40°, B = 12°, b = 100$

2. $C = 150°, a = 5, c = 20$

3. Find the area of the triangle: $a = 3, b = 6, C = 130°$.

4. Determine the number of solutions to the triangle: $a = 10, b = 35, A = 22.5°$.

For Exercises 5 and 6, use the Law of Cosines to find the remaining sides and angles of the triangle.

5. $a = 49, b = 53, c = 38$

6. $C = 29°, a = 100, c = 300$

7. Use Heron's Formula to find the area of the triangle: $a = 4.1, b = 6.8, c = 5.5$.

8. A ship travels 40 miles due east, then adjusts its course $12°$ southward. After traveling 70 miles in that direction, how far is the ship from its point of departure?

9. $\mathbf{w} = 4\mathbf{u} - 7\mathbf{v}$ where $\mathbf{u} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$. Find $\mathbf{w}$.

10. Find a unit vector in the direction of $\mathbf{v} = 5\mathbf{i} - 3\mathbf{j}$.

11. Find the dot product and the angle between $\mathbf{u} = 6\mathbf{i} + 5\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$.

12. $\mathbf{v}$ is a vector of magnitude 4 making an angle of $30°$ with the positive x-axis. Find $\mathbf{v}$ in component form.

13. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$ given $\mathbf{u} = \langle 3, -1 \rangle$ and $\mathbf{v} = \langle -2, 4 \rangle$.

14. Give the trigonometric form of $z = 5 - 5i$.

15. Give the standard form of $z = 6(\cos 225° + i \sin 225°)$.

16. Multiply $[7(\cos 23° + i \sin 23°)][4(\cos 7° + i \sin 7°)]$.

17. Divide $\dfrac{9\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)}{3(\cos \pi + i \sin \pi)}$.

18. Find $(2 + 2i)^8$.

19. Find the cube roots of $8\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$.

20. Find all the solutions to $x^4 + i = 0$.