THW1

① $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$, $x = -1 = f(3)$

$f^{-1}(-1) = 3$

$$\frac{df^{-1}}{dx} = \frac{1}{f'(f^{-1}(-1))} = \frac{1}{3(-1)^2 - 6(-1)} = \boxed{\frac{1}{9}}$$

② $f'(x) = \frac{1}{3}$ when $f(2) = 4$ or $f^{-1}(4) = 2$

$$\left.\frac{df^{-1}}{dx}\right|_{x=4} = \frac{1}{1/3} = \boxed{3}$$

③ Let f be an increasing function. Assume x_1 and x_2 are elements of the domain of f and $x_1 \neq x_2$. Then, $x_1 < x_2$ or $x_1 > x_2$, which implies $f(x_1) < f(x_2)$ or $f(x_1) > f(x_2)$, since $f(x)$ is increasing. Either case, $f(x_1) \neq f(x_2)$ and f is one-to-one. Similar arguments hold if f is decreasing.

④ For $x_1 < x_2$, $f(x_1) = (1-x_1)^3$ and $f(x_2) = (1-x_2)^3$.

Since $x_1 < x_2$, $1 - x_1 > 1 - x_2$ and $f(x_1) > f(x_2)$.

$\therefore$ f is decreasing throughout its domain, is a 1-1 function, and has an inverse function.

$f^{-1}(x) = 1 - \sqrt[3]{x_1}$ $\quad df^{-1}/dx = -\frac{1}{3\sqrt[3]{x_1^2}}$

⑤ $y = \ln(x^2+1)^5 - \ln(1-x)^{1/2}$

$y = 5\ln(x^2+1) - \frac{1}{2}\ln(1-x)$

$$\frac{dy}{dx} = \frac{5 \cdot 2x}{x^2+1} - \frac{-1}{2(1-x)}$$

$$\frac{dy}{dx} = \boxed{\frac{10x}{x^2+1} + \frac{1}{2-2x}}$$

⑥ $\int_2^4 \frac{dx}{x(\ln x)^2}$

$u = \ln x$

$du = \frac{1}{x}dx$

$[2, 4] \rightarrow [\ln 2, \ln 4]$

$$= \int_{\ln 2}^{\ln 4} \frac{du}{u^2}$$

$$= -\frac{1}{u}\Big|_{\ln 2}^{\ln 4} = -\left(\frac{1}{\ln 4} - \frac{1}{\ln 2}\right)$$

$$= \frac{1}{\ln 2} - \frac{1}{\ln 4} = \frac{1}{\ln 2} - \frac{1}{2\ln 2} = \frac{1}{2\ln 2}$$

$$= \boxed{\frac{1}{\ln 4}}$$

(7) $\displaystyle\int \frac{3\sec^2 t}{3(2+\tan t)}\,dt$ $\qquad u = 2+\tan t$, $\quad du = \sec^2 t\,dt$

$$= \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|2+\tan t| + C}$$

(8) $\displaystyle\int_0^{\pi/2} \tan\frac{x}{2}\,dx$ $\qquad u = \frac{x}{2}$, $\quad du = \frac{1}{2}dx$ $\qquad [0, \frac{\pi}{2}] \to [0, \frac{\pi}{4}]$

$$= 2\int_0^{\pi/4} \tan u\,du = 2\Big[\ln|\sec u|\Big]_0^{\pi/4} = 2\Big[\ln(\sec\tfrac{\pi}{4}) - \ln|\sec 0|\Big]$$

$$= 2\ln(\sqrt{2}) - 2\ln(1) = \boxed{\ln 2}$$

(9) $\displaystyle\int_{\pi/2}^{\pi} 2\cot\frac{\theta}{3}\,d\theta$ $\qquad u = \frac{\theta}{3}$, $\quad du = \frac{1}{3}d\theta$ $\qquad [\frac{\pi}{2}, \pi] \to [\frac{\pi}{6}, \frac{\pi}{3}]$

$$= 6\int_{\pi/6}^{\pi/3} \cot u\,du = -6\ln|\csc u|\Big|_{\pi/6}^{\pi/3} = -6\Big[\ln\Big|\frac{2\sqrt{3}}{3}\Big| - \ln 2\Big]$$

$$= 6\ln 2 - 6\ln\frac{2\sqrt{3}}{3} = \ln 64 - \ln\frac{64\cdot 3^3}{\cancel{729}_{27}} = \ln\frac{64}{\frac{64}{27}} = \boxed{\ln 27}$$

(10) $\displaystyle \ln y = \ln\frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}} = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$

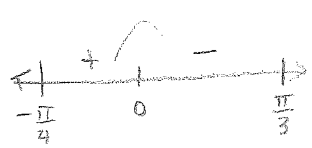
$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} + \frac{\cancel{2}x}{\cancel{2}(x^2+1)} - \frac{2}{3(x+1)}$$

$$\boxed{\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right)}$$

(11) a) $y = \ln(\cos x)$ $\qquad [-\frac{\pi}{4}, \frac{\pi}{3}]$

$y' = \frac{-\sin x}{\cos x} = -\tan x$

$y' = 0 \to x = 0, \pm\pi, \pm 2\pi, \ldots$



x	$-\pi/4$	0	$\pi/3$
y	$\ln(\frac{1}{\sqrt{2}})$	0	$\ln\frac{1}{2}$
	$-\frac{1}{2}\ln 2$	max	$-\ln 2$ min

abs min: $f(\frac{\pi}{3}) = -\ln 2$

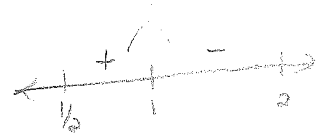
abs max: $f(0) = 0$

b) $y = \cos(\ln x)$ $\qquad [\frac{1}{2}, 2]$

$y' = -\sin(\ln x)\cdot\frac{1}{x} = \frac{-\sin(\ln x)}{x}$

$y' = 0 \to x = 1$

y' is und $\to$ ~~$x = 0$~~



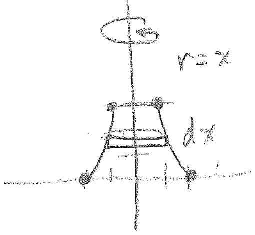
x	$1/2$	1	2
y	$\cos(-\ln 2)$ $\cos(\ln 2)$	1	$\cos\ln 2$ $\cos(\ln 2)$

abs min: at $x = \frac{1}{2}$ and $x = 2$

abs max: at $x = 1$

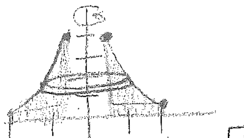
(12) $\displaystyle\int_1^5 (\ln 2x - \ln x)\,dx = \int_1^5 \ln 2\,dx = x\ln 2\Big|_1^5 = 5\ln 2 - \ln 2 = \boxed{\ln 16}$

(13)



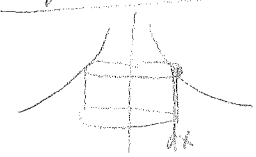
$$V = \pi\int_0^3 \frac{4}{y+1}\,dy = 4\pi\Big[\ln|y+1|\Big]_0^3 = 4\pi(\ln 4 - \ln 1)$$
$$= \boxed{4\pi\ln 4}$$

(14)



$y = \frac{1}{x^2}$ $\quad R = x = \sqrt{\frac{1}{y}}$

$x^2 = \frac{1}{y}$ $\quad r = 1/2$



$r = x,\ h = y = \frac{1}{x^2}$

$V = \pi\int_0^4 \left(\frac{1}{y} - \frac{1}{4}\right)dy \to$ can't be done $\to$ try shells

$$V = 2\pi\int_{1/2}^2 x\left(\frac{1}{x^2}\right)dx = 2\pi\int_{1/2}^2 \frac{1}{x}\,dx$$
$$V = 2\pi\Big[\ln x\Big]_{1/2}^2 = 2\pi\left(\ln 2 - \ln\frac{1}{2}\right)$$
$$= 2\pi(2\ln 2)$$
$$= \boxed{4\pi\ln 2}$$

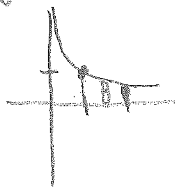
(15) a) $y = \frac{x^2}{8} - \ln x,\ 4 \le x \le 8$

$$y' = \frac{x}{4} - \frac{1}{x} = \frac{x^2-4}{4x}$$
$$1 + (y')^2 = 1 + \frac{x^4 - 8x^2 + 16}{16x^2}$$
$$= \frac{x^4 + 8x^2 + 16}{16x^2} = \frac{(x^2+4)^2}{(4x)^2}$$
$$S = \int_4^8 \sqrt{\left(\frac{x^2+4}{4x}\right)^2}\,dx = \int_4^8 \frac{x^2+4}{4x}\,dx$$
$$S = \int_4^8 \frac{x}{4} + \frac{1}{x}\,dx = \frac{x^2}{8}\Big|_4^8 + \ln x\Big|_4^8$$
$$= (8-2) + (\ln 8 - \ln 4)$$
$$\boxed{S = 6 + \ln 2}$$

b) $x = \left(\frac{y}{4}\right)^2 - 2\ln\left(\frac{y}{4}\right),\ 4 \le y \le 12$

$$x' = \frac{y}{8} - \frac{2}{y/4}\cdot\cancel{\frac{1}{4}}$$
$$x' = \frac{y}{8} - \frac{2}{y} = \frac{y^2-16}{8y}$$
$$1 + (x')^2 = 1 + \frac{y^4 - 32y^2 + 256}{64y^2}$$
$$= \frac{y^4 + 32y^2 + 256}{64y^2} = \left(\frac{(y^2+16)^2}{64y^2}\right)$$
$$S = \int_4^{12} \frac{y^2+16}{8y}\,dy = \int_4^{12} \frac{y}{8} + \frac{2}{y}\,dy$$
$$S = \frac{y^2}{16}\Big|_4^{12} + 2\ln|y|\Big|_4^{12} = 9 - 1 + 2\ln 12 - 2\ln 4$$
$$\boxed{S = 8 + 2\ln 3}$$

(16) $y=\frac{1}{x}$



$m=\rho\int_1^2 \frac{1}{x}\,dx = \rho\left[\ln x\right]_1^2 = \rho\ln 2$

$M_y=\rho\int_1^2 x\left(\frac{1}{x}\right)dx = \rho x\Big|_1^2 = \rho$

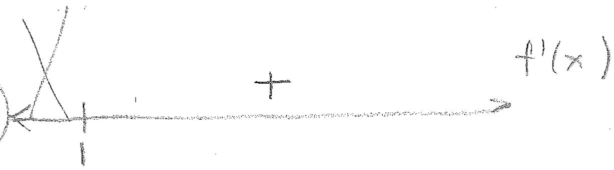
$M_x=\rho\int_1^2 \frac{1}{2}\left(\frac{1}{x}\right)\left(\frac{1}{x}\right)dx = \frac{\rho}{2}\int_1^2 \frac{1}{x^2}\,dx = -\frac{\rho}{2}\left[\frac{1}{x}\right]_1^2 = -\frac{\rho}{2}\left(\frac{1}{2}-1\right) = \frac{\rho}{4}$

Centroid: $\left(\frac{\rho}{\rho\ln 2}, \frac{\rho/4}{\rho\ln 2}\right) = \boxed{\left(\frac{1}{\ln 2}, \frac{1}{4\ln 2}\right)}$

(17) $f(x)=\ln(x^3-1)$ Since $x^3-1>0$, $x>1$.

$f'(x)=\frac{1}{x^3-1}\cdot 3x^2$

$f'(x)=\frac{3x^2}{x^3-1}$, $f'(x)=0$
$x=0 \notin (1,\infty)$



$f'(x)>0$ on the domain $(1,\infty)$.

So, $f(x)$ is increasing throughout its domain $(1,\infty)$.

$\therefore$ f is one-to-one.

(18) $\frac{dy}{dx}=1+\frac{1}{x}$, $y(1)=3$

$y=x+\ln|x|+C$

$3=1+0+C$

$C=2$

$\boxed{y=x+\ln|x|+2}$

(19) $\ln y = e^y\sin x$

$\frac{d}{dx}(\ln y)=\frac{d}{dx}(e^y\sin x)$

$\frac{1}{y}\cdot\frac{dy}{dx}=e^y\cdot\frac{dy}{dx}\sin x+e^y\cos x$

$\frac{1}{y}\frac{dy}{dx}-e^y\sin x\cdot\frac{dy}{dx}=e^y\cos x$

$\frac{dy}{dx}\left(\frac{1}{y}-e^y\sin x\right)=e^y\cos x$

$\boxed{\frac{dy}{dx}=\frac{e^y\cos x}{\frac{1}{y}-e^y\sin x}}$

(20) $\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\,dr$ $\quad u=\sqrt{r}$, $du=\frac{1}{2\sqrt{r}}\,dr$

$= 2\int e^u\,du = 2e^u + C = \boxed{2e^{\sqrt{r}} + C}$

(21) $\int e^{\sec \pi t}\sec \pi t \tan \pi t \, dt$ $\quad u=\sec \pi t$, $du = \sec \pi t \tan \pi t \cdot \pi \, dt$

$= \frac{1}{\pi}\int e^u\,du = \frac{e^u}{\pi} + C = \boxed{\frac{e^{\sec \pi t}}{\pi} + C}$

(22) $\frac{d^2y}{dx^2} = 2e^{-x}$, $y(0)=1$; $y'(0)=0$

$\frac{dy}{dx} = \int 2e^{-x}dx = -2e^{-x} + C$

$0 = -2e^0 + C \rightarrow C = 2$

$\frac{dy}{dx} = -2e^{-x} + 2$

$y = \int(-2e^{-x}+2)\,dx = 2e^{-x} + 2x + C$

$1 = 2e^0 + 0 + C \rightarrow C = -1$

$\boxed{y = 2e^{-x} + 2x - 1}$

(23) $y = (\cos\theta)^{\sqrt{2}}$

$\frac{dy}{d\theta} = \sqrt{2}(\cos\theta)^{\sqrt{2}-1}(-\sin\theta) = \boxed{-\sqrt{2}\sin\theta(\cos\theta)^{\sqrt{2}-1}}$

(24) $y = 7^{\sec\theta}\ln 7$

$\frac{dy}{d\theta} = 7^{\sec\theta}\cdot \ln 7 \cdot \sec\theta\tan\theta \cdot \ln 7$

$\boxed{\frac{dy}{d\theta} = 7^{\sec\theta}(\ln 7)^2 \sec\theta\tan\theta}$

(25) $y = \log_4 x + \log_4 x^2$

$$\frac{dy}{dx} = \frac{1}{x\ln 4} + \frac{2x}{x^2 \ln 4} = \boxed{\frac{3}{x\ln 4}}$$

(26) $y = \ln 3 \log_3\left(\frac{x+1}{x-1}\right) = \ln 3\left[\log_3(x+1) - \log_3(x-1)\right]$

$$\frac{dy}{dx} = \ln 3\left(\frac{1}{(x+1)\ln 3} - \frac{1}{(x-1)\ln 3}\right) = \frac{1}{x+1} - \frac{1}{x-1} = \frac{x-1-x-1}{x^2-1} = \boxed{\frac{-2}{x^2-1}}$$

(27) $y = \theta \sin(\log_7 \theta)$

$$\frac{dy}{d\theta} = \sin(\log_7 \theta) + \theta\cos(\log_7\theta) \cdot \frac{1}{\theta \ln 7}$$

$$\frac{dy}{d\theta} = \boxed{\sin(\log_7\theta) + \frac{\cos(\log_7\theta)}{\ln 7}}$$

(28) $y = \log_2(8t^{\ln 2})$

$$\frac{dy}{dt} = \frac{1}{\cancel{8}t^{\ln 2} \cdot \cancel{\ln 2}} \cdot \cancel{8 \ln 2}\, t^{\ln 2 - 1} = \boxed{\frac{1}{t}}$$

(29) $\displaystyle\int_0^2 \frac{\log_2(x+2)}{x+2}\,dx$ $\quad u = \log_2(x+2)$ $\quad du = \frac{1}{(x+2)\ln 2}\,dx$ $\quad [0,2] \to [1, 2]$

$$= \int_1^2 (\ln 2)\, u\, du = \frac{\ln 2}{2} u^2 \Big|_1^2 = \boxed{\frac{3}{2}\ln 2}$$

(30) $\displaystyle\int_1^{1/x} \frac{1}{t}\,dt,\ x > 0$

$$= \Big[\ln|t|\Big]_1^{1/x} = \ln\frac{1}{x} - \ln 1 = \boxed{-\ln x}$$

THW2

(31) $y = \sin x^{x}$

$z = x^{x}$

$\ln z = x \ln x$

$\frac{1}{z} \cdot \frac{dz}{dx} = \ln x + 1$

$\frac{dz}{dx} = x^{x}(\ln x + 1)$

$\frac{dy}{dx} = \cos x^{x} \cdot \frac{d}{dx}(x^{x})$

$\boxed{\frac{dy}{dx} = x^{x} \cos x^{x} (\ln x + 1)}$

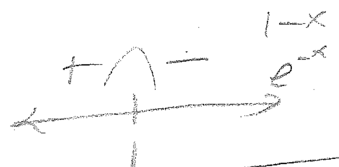
(32) $f(x) = xe^{-x}$

a) $f'(x) = e^{-x} - xe^{-x}$

$f'(x) = e^{-x}(1-x)$

$f'(x) = 0$

$x = 1$

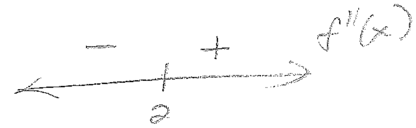


$\boxed{\text{abs max: } f(1) = \frac{1}{e}}$

b) $f''(x) = -e^{-x} - e^{-x} + xe^{-x}$

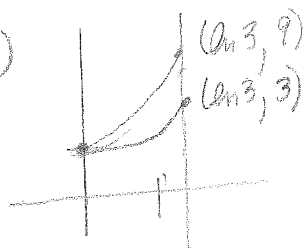
$f''(x) = -2e^{-x} + xe^{-x}$

$f''(x) = 0 \rightarrow \quad x = 2$



$\boxed{\text{inf pt}: \left(2, \frac{2}{e^2}\right)}$

(33)



$A = \int_0^{\ln 3} (e^{2x} - e^{x})\,dx = \left[\frac{1}{2}e^{2x} - e^{x}\right]_0^{\ln 3}$

$= \left(\frac{1}{2} \cdot 9 - 3\right) - \left(\frac{1}{2} - 1\right) = \frac{3}{2} + \frac{1}{2} = \boxed{2}$

(34) $[f'(x)]^2 = \frac{1}{4}e^{x}$; origin $\rightarrow f(0) = 0$

$f'(x) = \frac{1}{2}e^{\frac{1}{2}x}$

$f(x) = \int \frac{1}{2}e^{\frac{1}{2}x}\,dx = \frac{\frac{1}{2}}{\frac{1}{2}} e^{\frac{1}{2}x} + C$

$f(0) = 0 \rightarrow 0 = e^{\frac{0}{2}} + C \rightarrow C = -1$

$\boxed{f(x) = e^{x/2} - 1}$

(35) $y = \frac{1}{2}(e^x + e^{-x})$ $x = 0$ to $x = 1$

$$\frac{dy}{dx} = \frac{1}{2}(e^x - e^{-x})$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{e^{2x} - 2 + e^{-2x}}{4}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4} = \frac{1}{4}(e^x + e^{-x})^2$$

$$s = \int_0^1 \sqrt{\frac{1}{4}(e^x + e^{-x})^2}\, dx = \int_0^1 \frac{1}{2}(e^x + e^{-x})\, dx = \frac{1}{2}\left[e^x - e^{-x}\right]_0^1$$

$$s = \frac{1}{2}\left[\left(e - \frac{1}{e}\right) - (1 - 1)\right] = \boxed{\frac{1}{2}\left(e - \frac{1}{e}\right)}$$

(36) a) $\int \ln x\, dx = x \ln x - x + C$

$$\frac{d}{dx}(x \ln x - x + C) = \ln x + x\left(\frac{1}{x}\right) - 1 + 0 = \ln x$$

b) $$\frac{1}{e-1}\int_1^e \ln x\, dx = \frac{1}{e-1}\left[x \ln x - x\right]_1^e = \frac{1}{e-1}\left[(e - e) - (0 - 1)\right]$$

$$= \boxed{\frac{1}{e-1}}$$

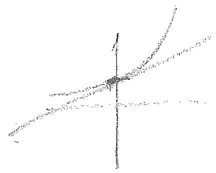
(37) $f(x) = e^x$
$y = e^x$ at $x = 0$

a) $y' = e^x$; $\left.\frac{dy}{dx}\right|_{x=0} = 1$ $y - 1 = 1(x - 0) \rightarrow y = x + 1$

$\boxed{L(x) = x + 1}$

b) $f(0) = 1$ and $L(0) = 1$; error $= 0$

$f(0.2) = e^{0.2} \doteq 1.22140$ and $L(0.2) = 1.2$; error $= \boxed{0.02140}$

c) 

$\boxed{L(x) \text{ only underestimates } f(x).}$